Robotics and Embedded Systems

Technische Universität München

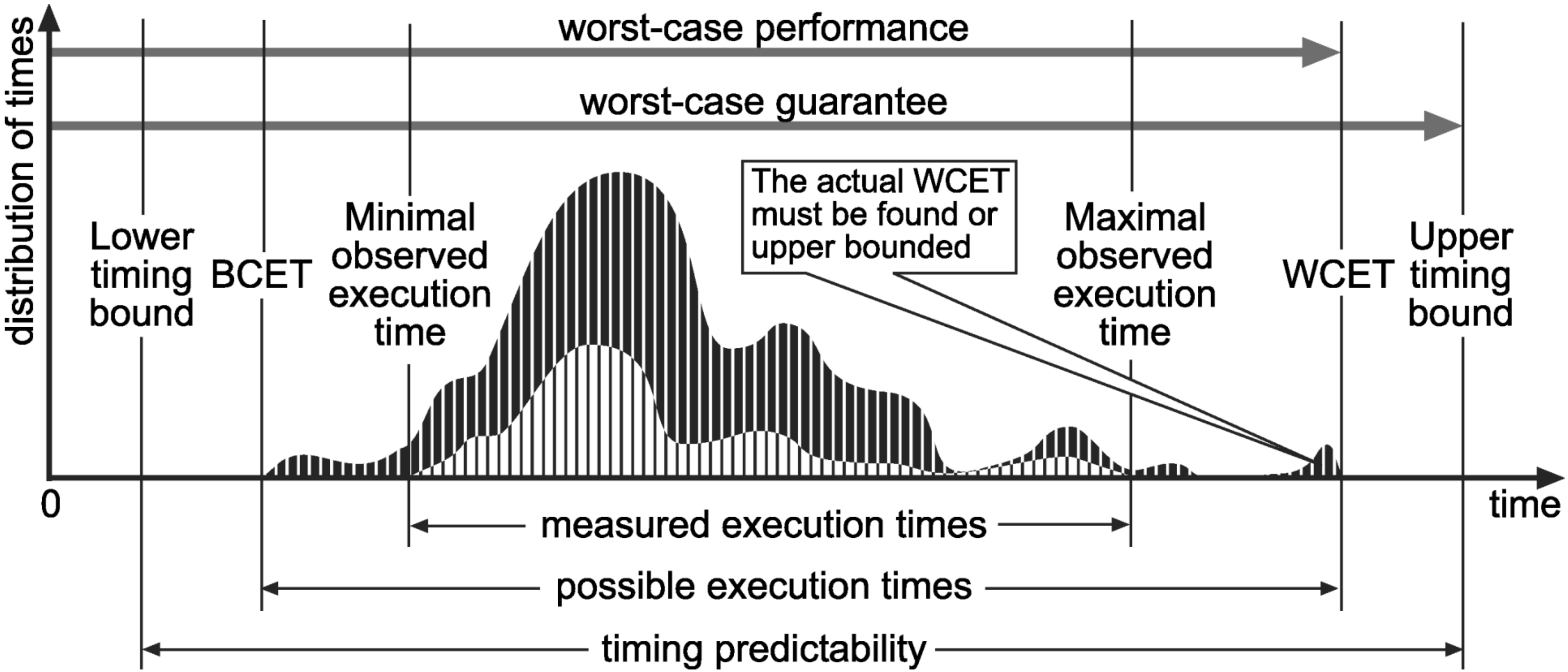
Analysis (II)
Modular Performance Analysis
(Real-Time Calculus)

Kai Huang

Outline

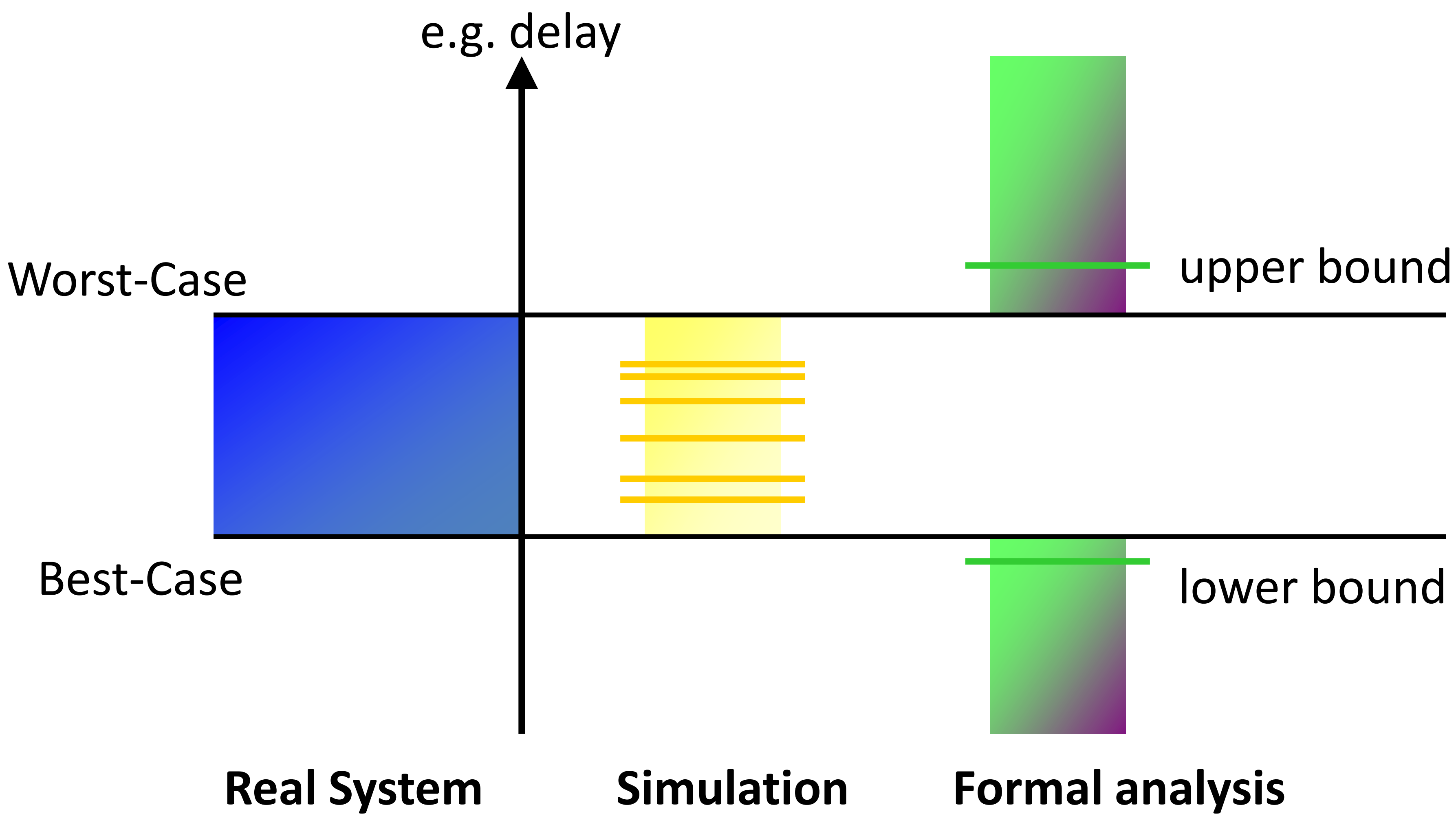
- Timing analysis in general
- Real-Time Calculus
- MPA

Timing Analysis Overview



R. Wilhelm, J. Engblom, A. Ermedahl, N. Holsti, S. Thesing, D. Whalley, G. Bernat, C. Ferdinand, R. Heckmann, T. Mitra, F. Mueller, I. Puaut, P. Puschner, J. Staschulat, and P. Stenstr ¨om. The worst-case execution-time problem—overview of methods and survey of tools. ACM Trans. Embed. Comput. Syst., 7(3):1–53, 2008

Formal Analysis vs. Simulation

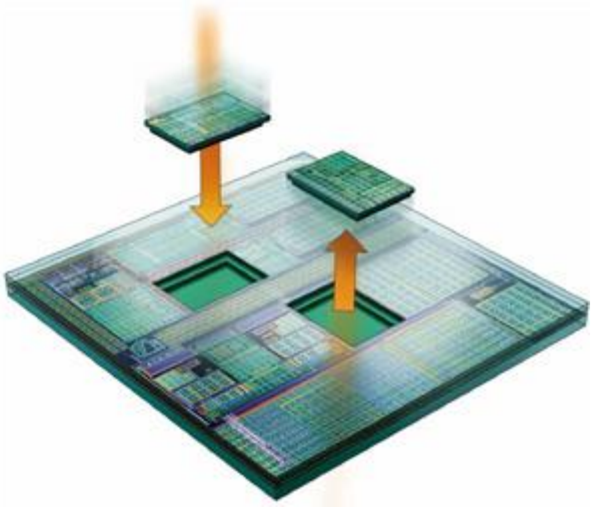


Analysis and Design

Embedded System = Computation
+ Communication
+ Resource Interaction

Analysis:

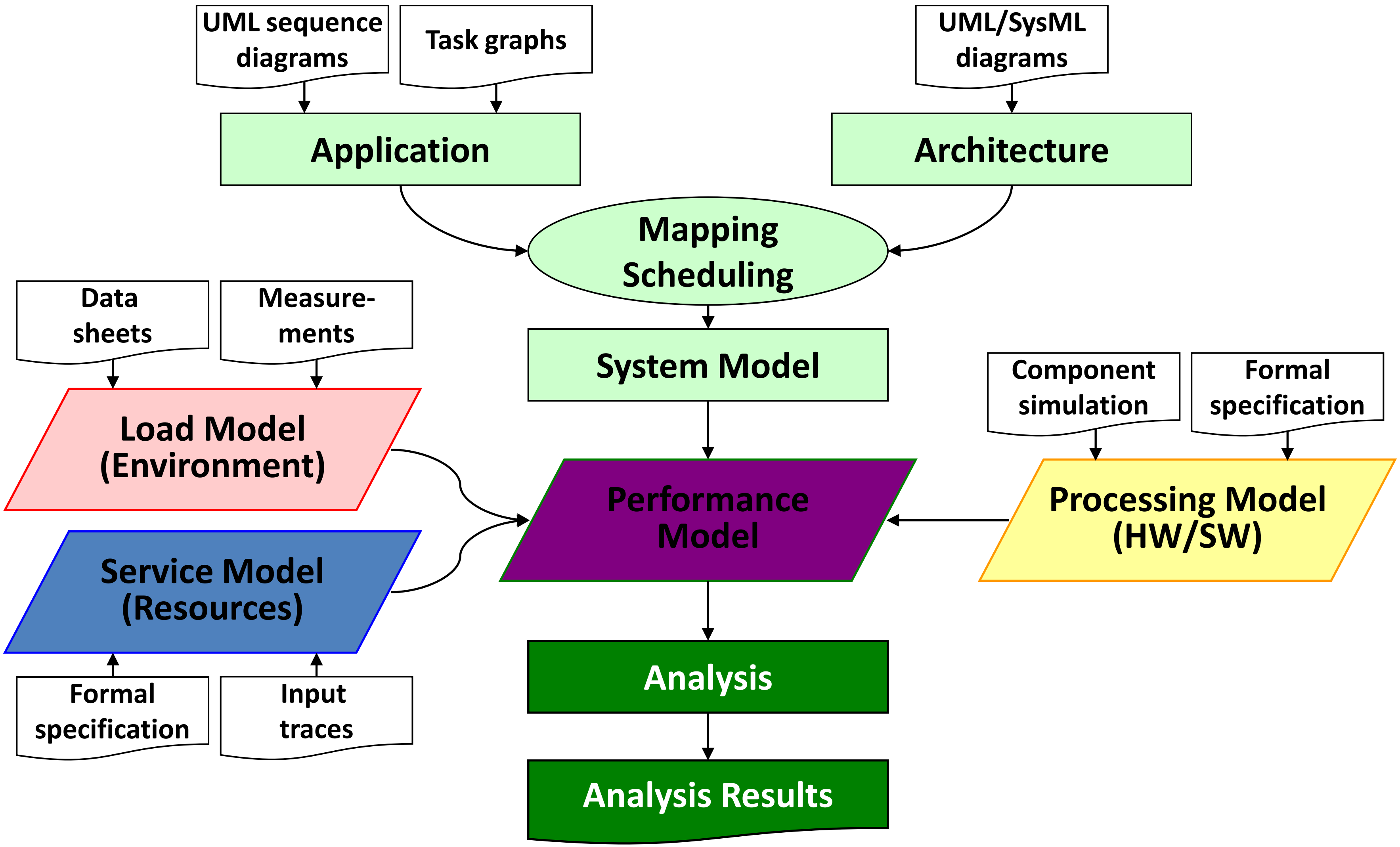
Infer system properties from subsystem properties.



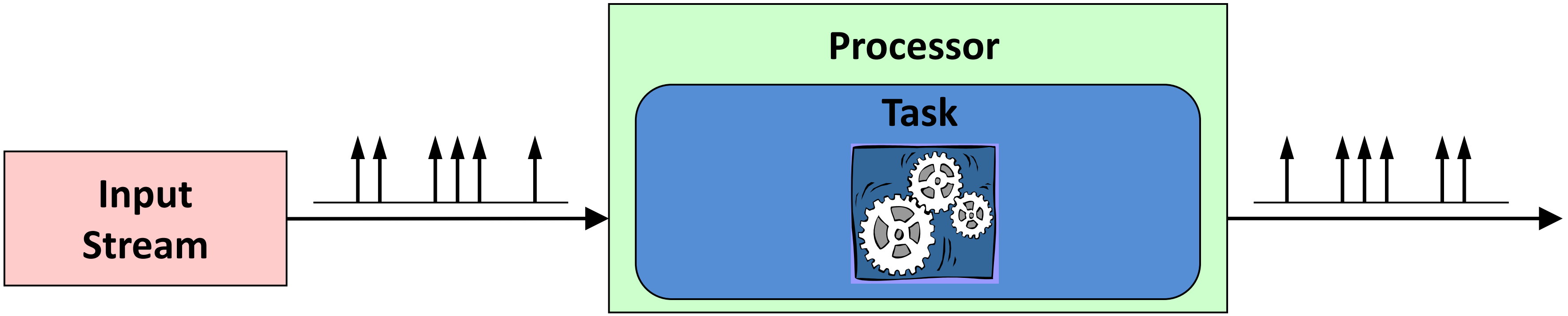
Design:

Build a system from subsysten while meeting requirements.

Modular Performance Analysis

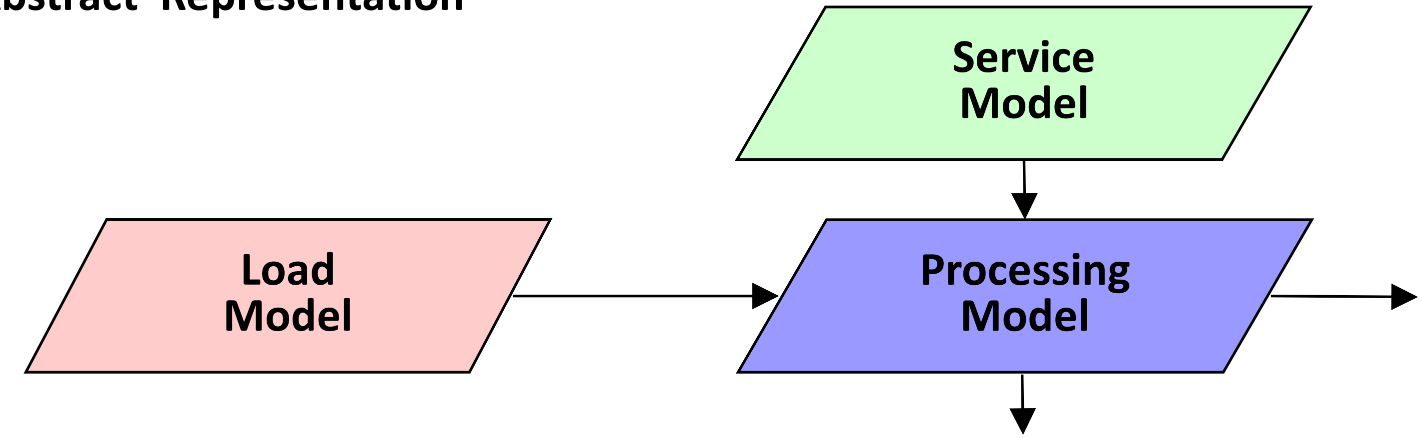


Abstract Models for Performance Analysis

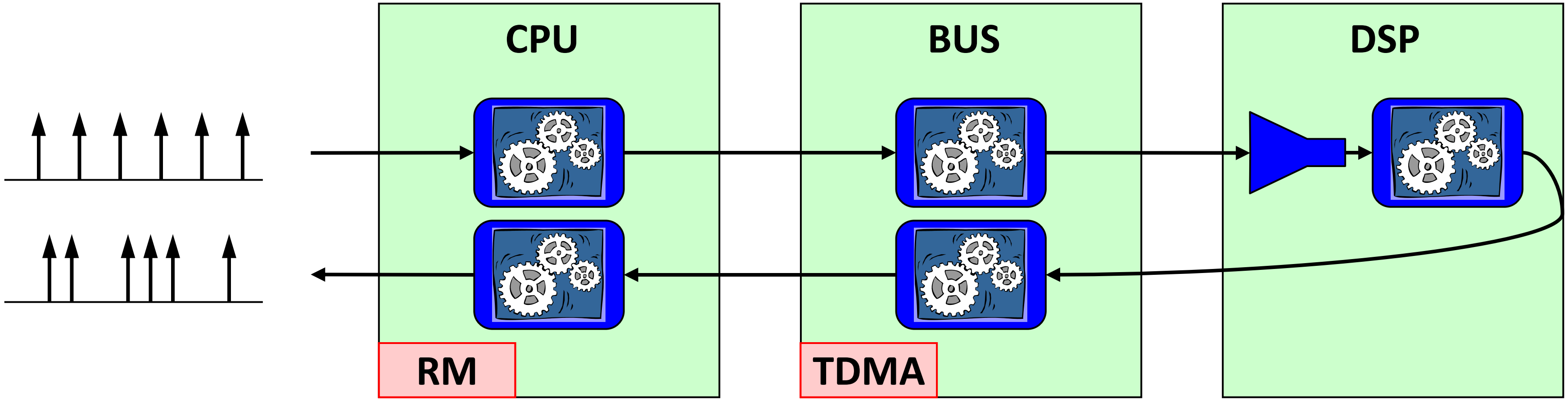


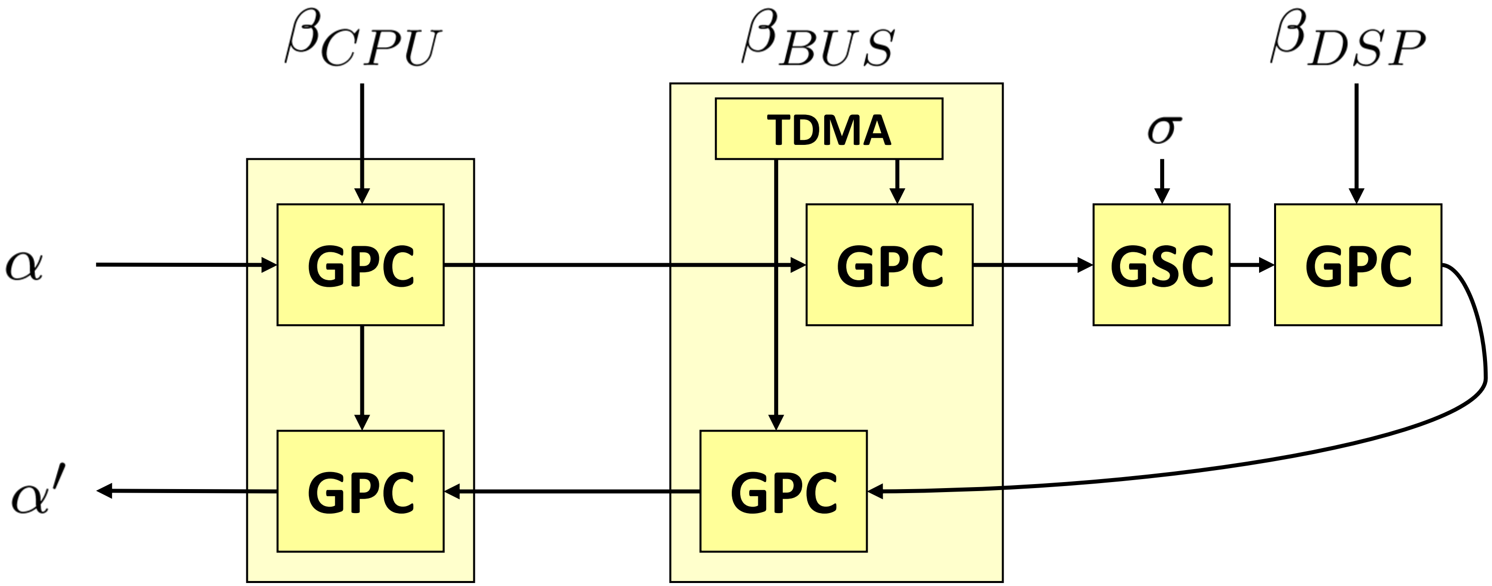
Concrete Instance

Abstract Representation

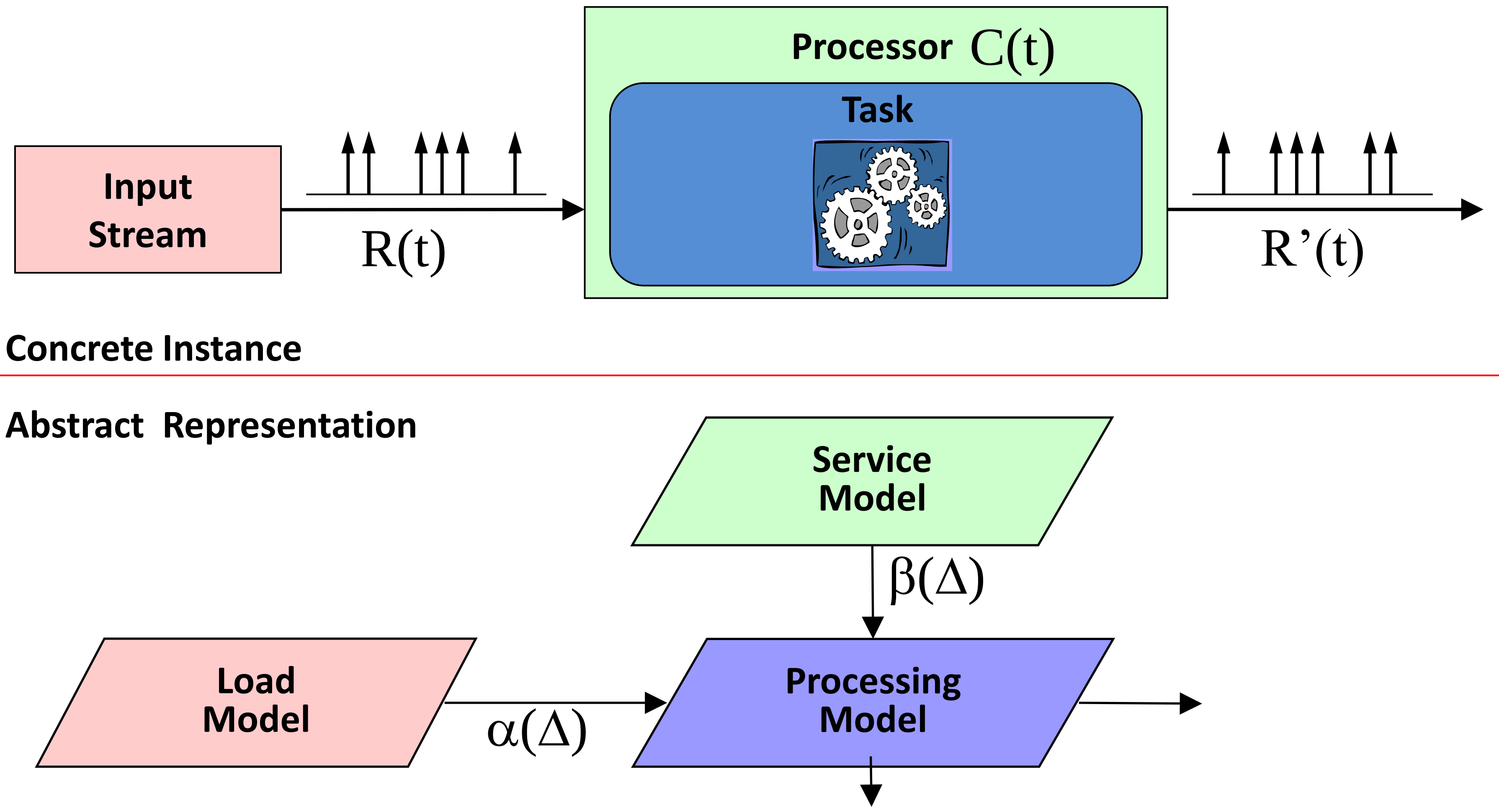


Modular System Composition





Abstract Models for Performance Analysis



Outline

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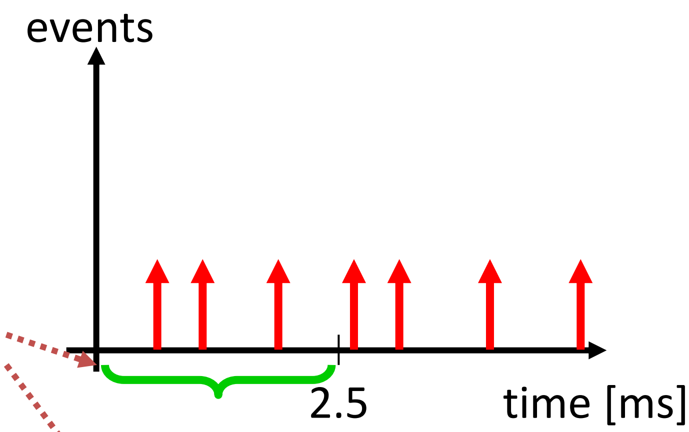
Event Stream Model: Arrival Curves

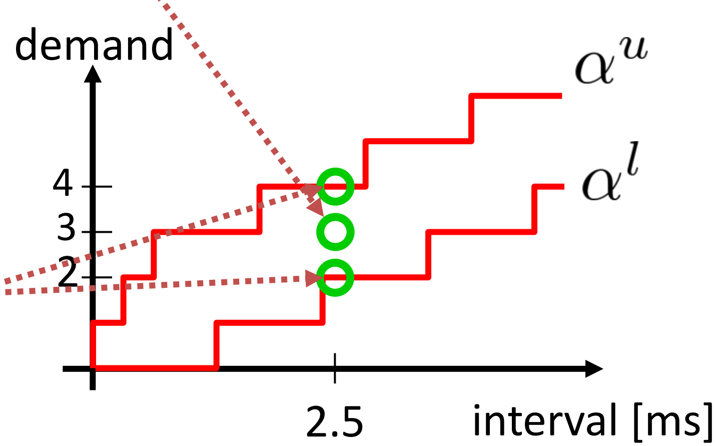
Event Stream

number of events in
in t=[0 .. 2.5] ms



Arrival Curve $[\alpha^l, \ \alpha^u]$

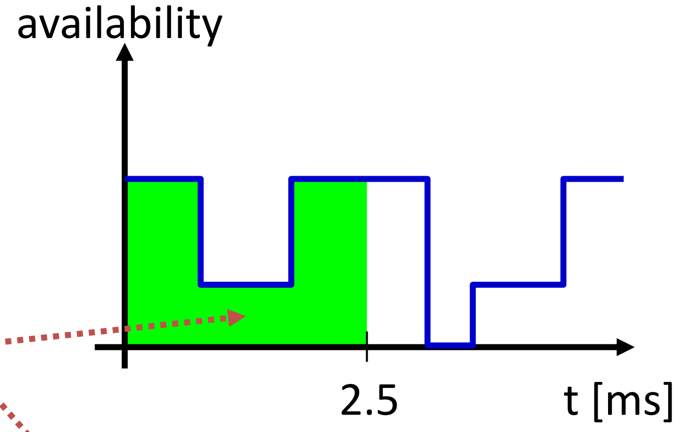
maximum / minimum
arriving demand in *any*
interval of length 2.5 ms



Service Model (Resources)

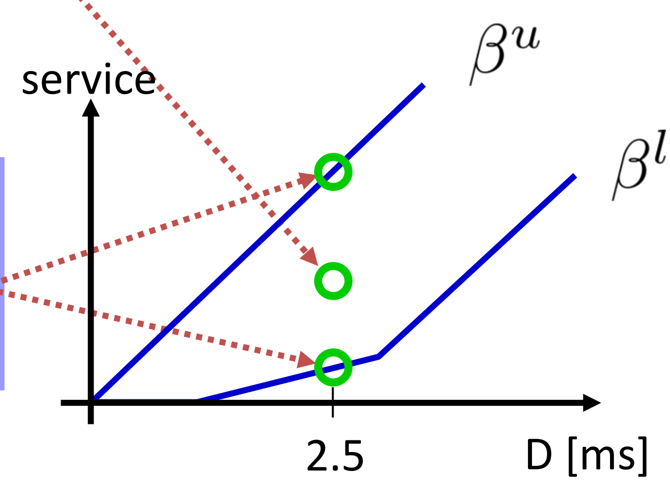
Resource Availability

available service
in t=[0 .. 2.5] ms



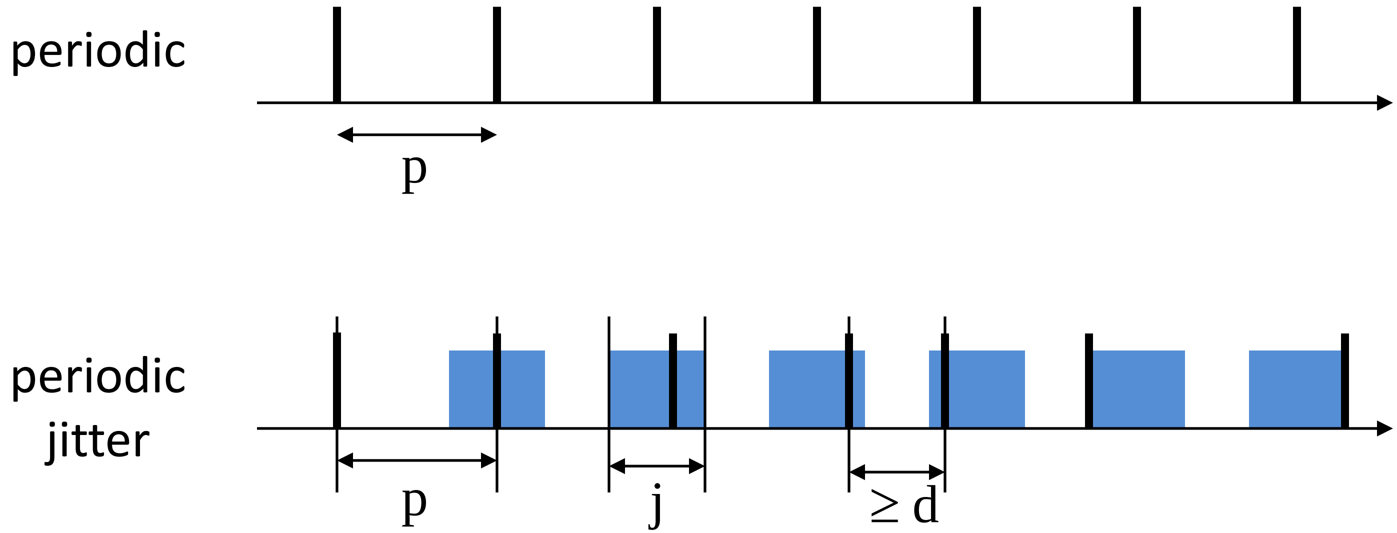
Service Curves $[\beta^l, \beta^u]$

maximum/minimum
available service in *any*
interval of length 2.5 ms

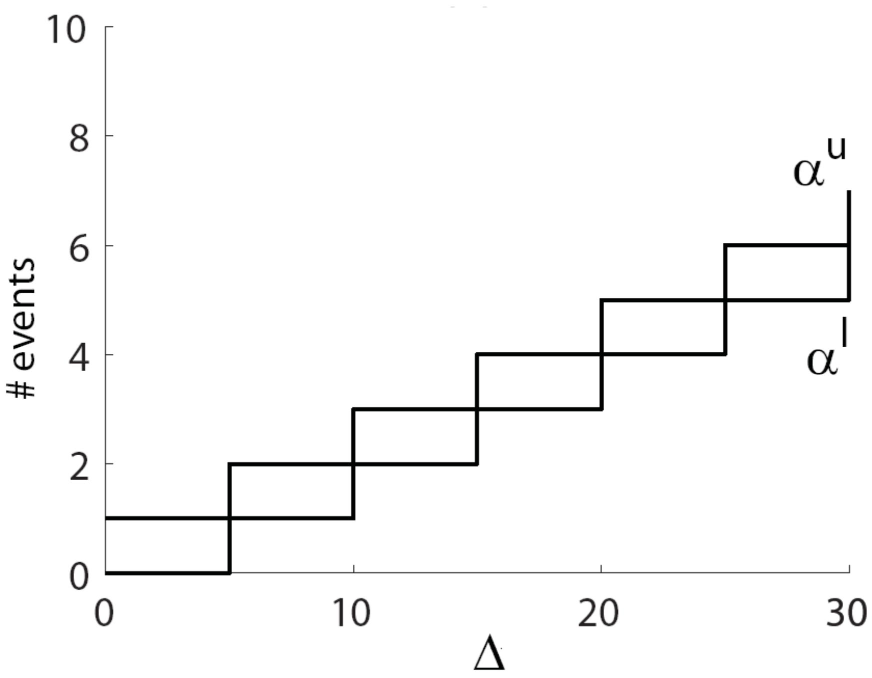


Example 1: Event Arrival Model

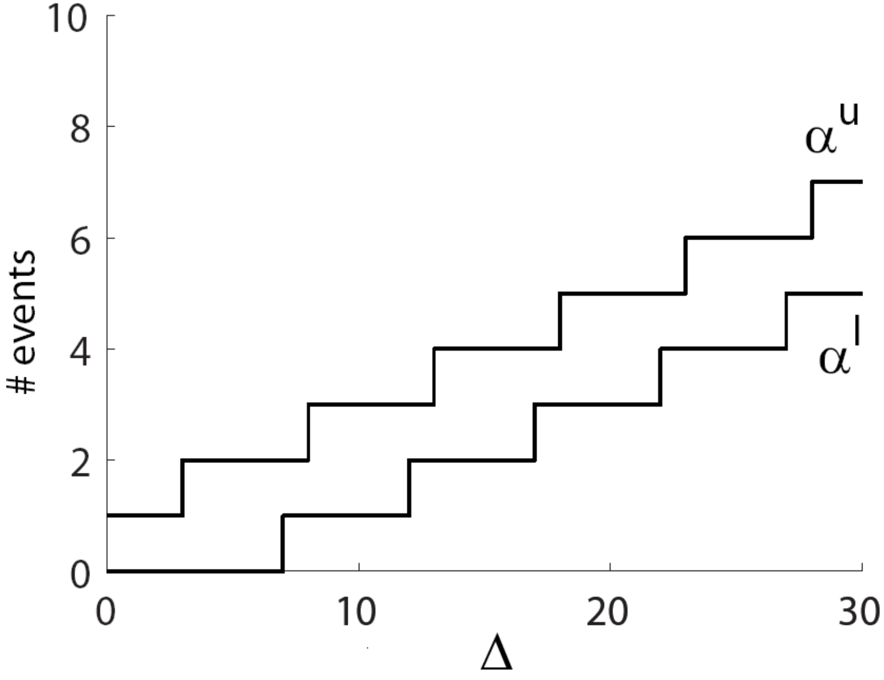
- A ***common event pattern*** that is used in literature can be specified by the parameter triple (*p*, *j*, *d*), where *p* denotes the period, *j* the jitter, and *d* the minimum inter-arrival distance of events in the modeled stream.



Example 1: Periodic (with Jitter)

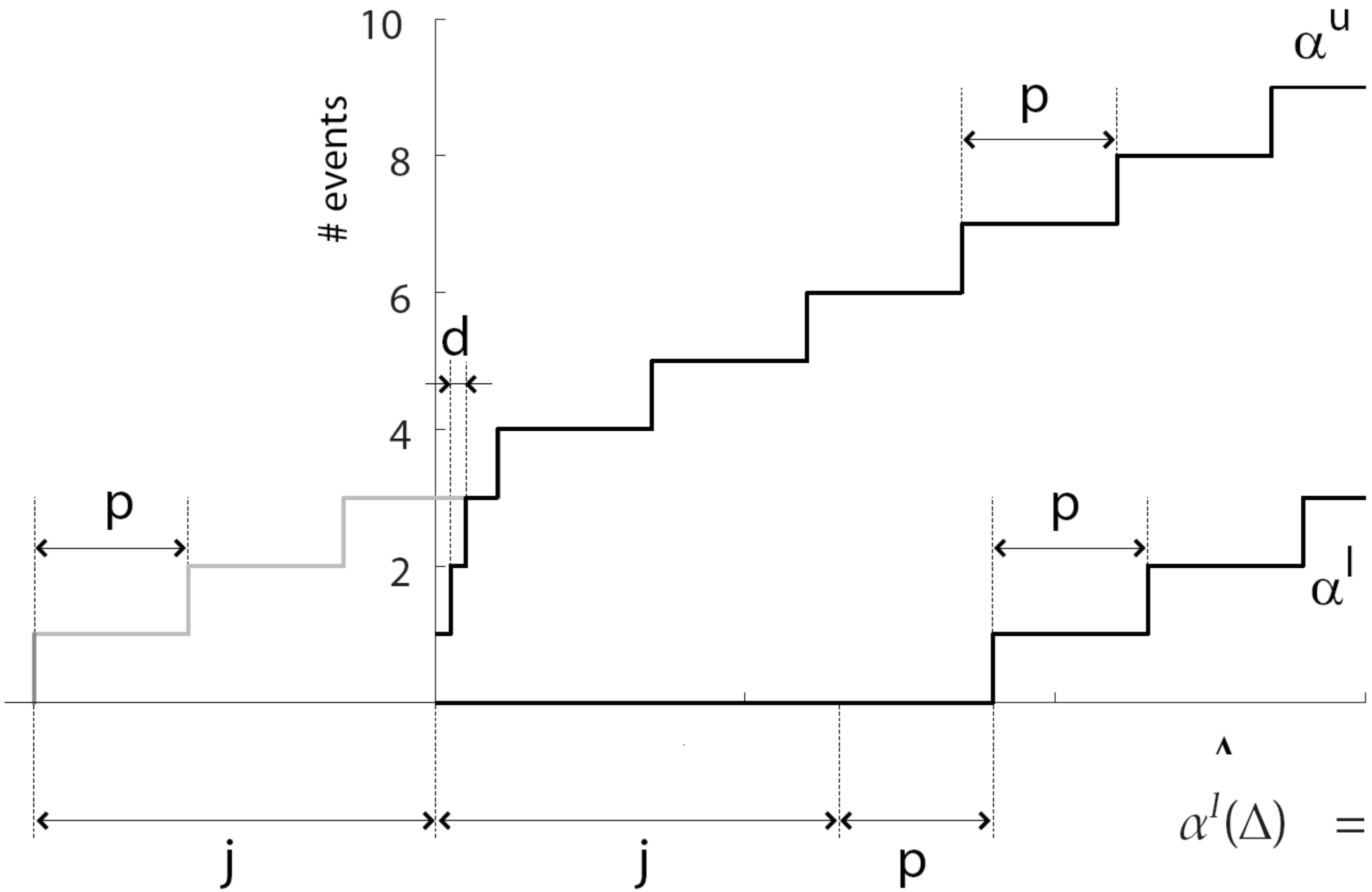


periodic



periodic with jitter

Example 1: (p, J, d) model

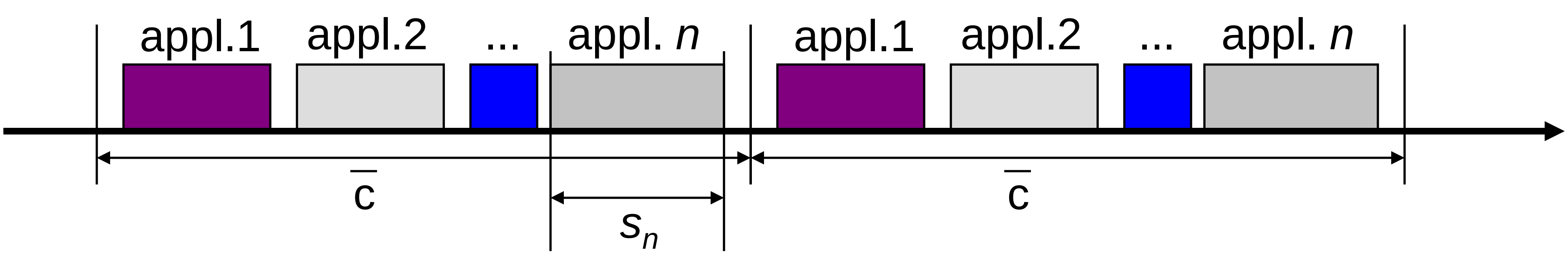


$$\alpha^l(\Delta) = \left\lfloor \frac{\Delta - j}{p} \right\rfloor$$

$$\alpha^u(\Delta) = \min\left\{ \left\lceil \frac{\Delta + j}{p} \right\rceil, \left\lceil \frac{\Delta}{d} \right\rceil \right\}$$

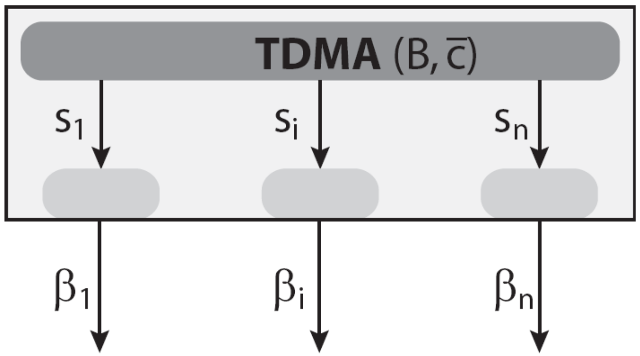
Example 2: TDMA Resource

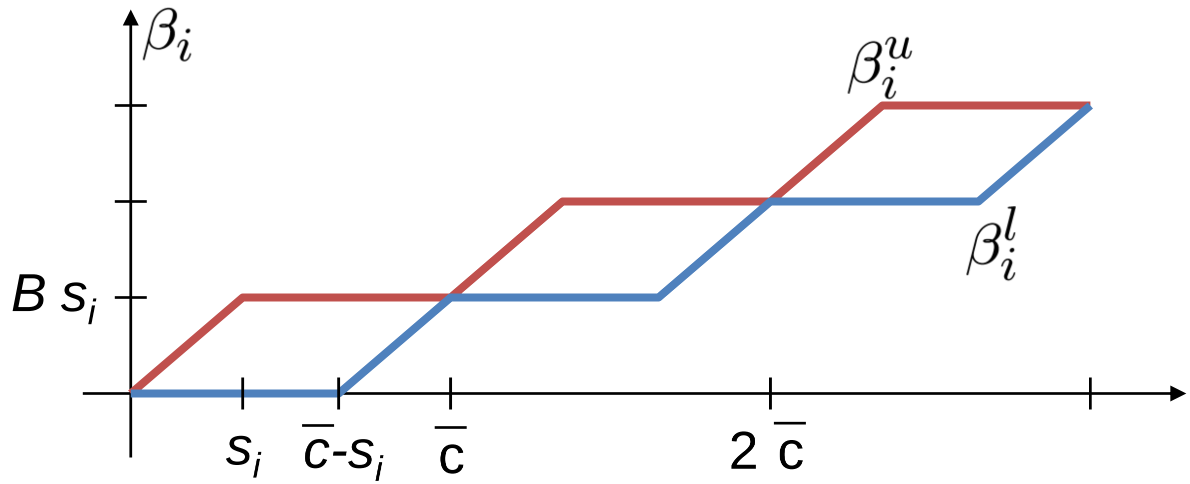
- Consider a real-time system consisting of *n* ***applications*** that are executed on a resource with bandwidth *B* that controls resource access using a ***TDMA policy***.
- Analogously, we could consider a distributed system with *n* ***communicating nodes***, that communicate via a shared bus with bandwidth *B*, with a bus arbitrator that implements a TDMA policy.
- ***TDMA policy***: In every TDMA cycle of length , one single resource slot of length s_i is assigned to application *i*.



Example 2: TDMA Resource

- ***Service curves*** available to the applications / node *i:*



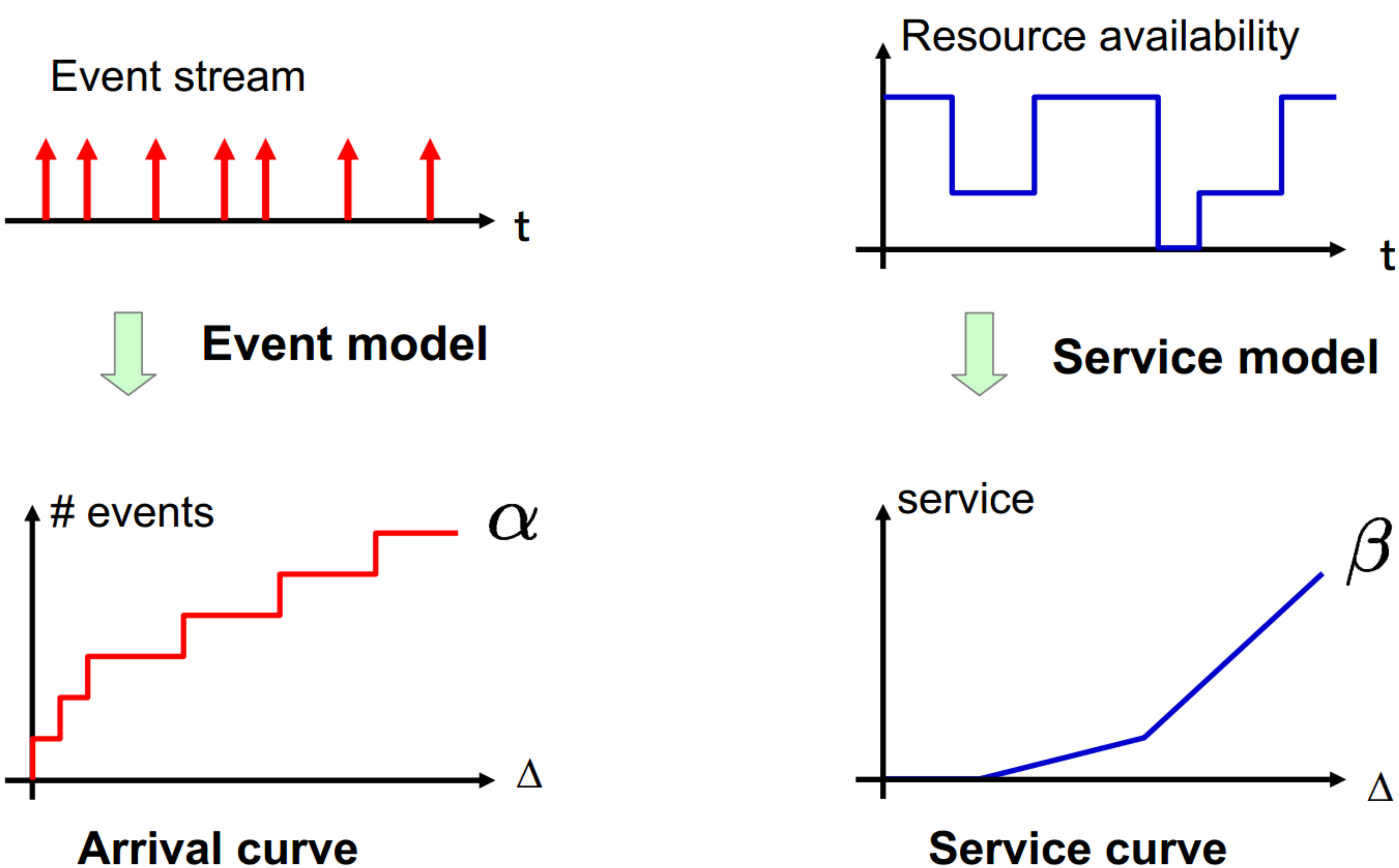


$$\beta_i^l(\Delta) = B\max\{\left\lfloor \frac{\Delta}{\bar{c}} \right\rfloor s_i, \Delta - \left\lceil \frac{\Delta}{\bar{c}} \right\rceil (\bar{c} - s_i)\}$$

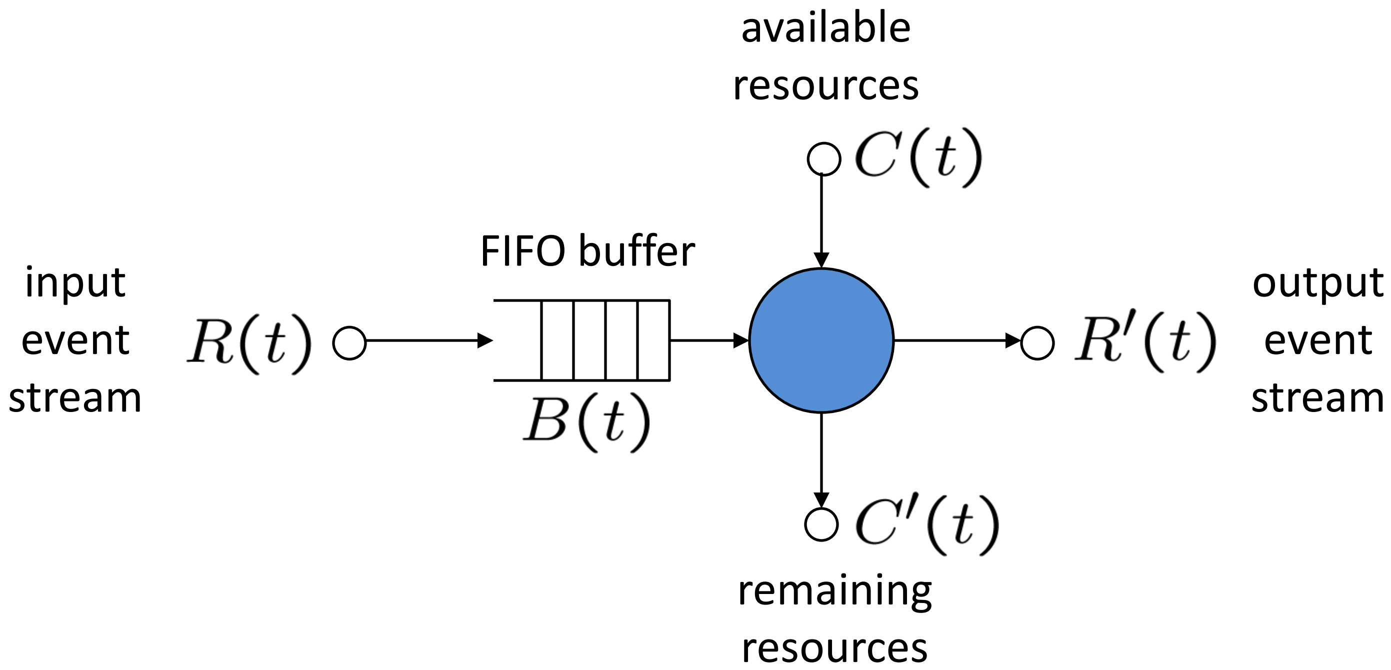
$$\beta_i^u(\Delta) = B\min\{\left\lceil \frac{\Delta}{\bar{c}} \right\rceil s_i, \Delta - \left\lfloor \frac{\Delta}{\bar{c}} \right\rfloor (\bar{c} - s_i)\}$$

Event and Resource Models

Real-Time Calculus (RTC) [Thiele et al. 2000]

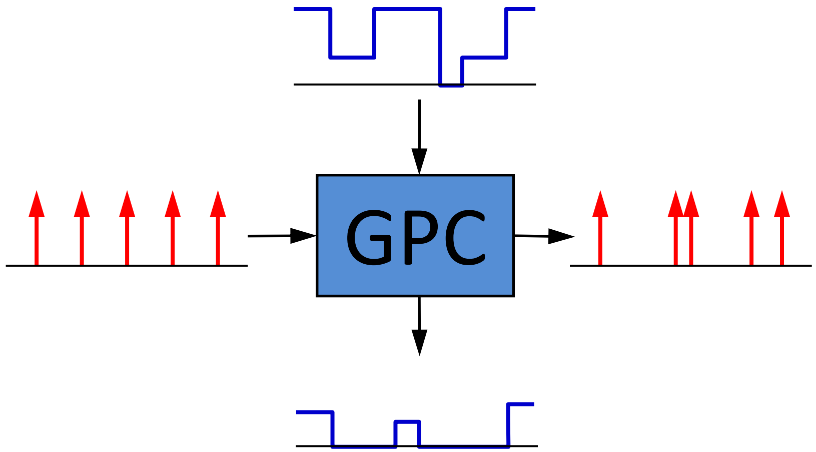


Greedy Processing Component (GPC)



- ***Examples:***
  - computation (event – task instance, resource – computing resource [tasks/second])
  - communication (event – data packet, resource – bandwidth [packets/second])

Greedy Processing Component



- Behavioral Description
  - Component is triggered by incoming events.
  - A fully preemptable task is instantiated at every event arrival to process the incoming event.
  - Active tasks are processed in a greedy fashion in FIFO order.
  - Processing is restricted by the availability of resources.

Greedy Processing Component (GPC)

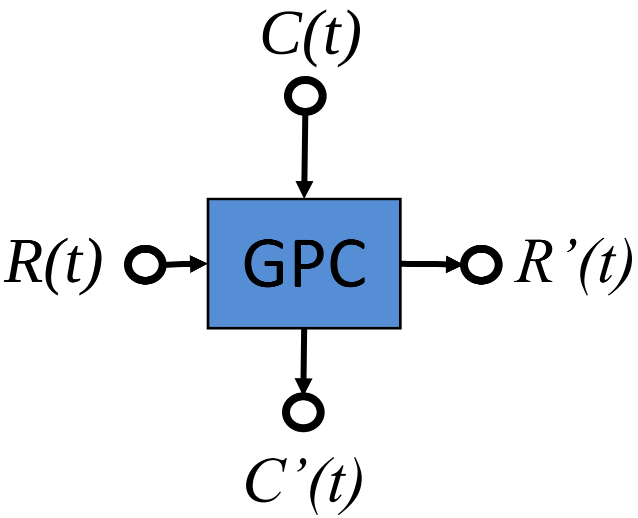
If the resource and event streams describe available and requested units of processing or communication, then

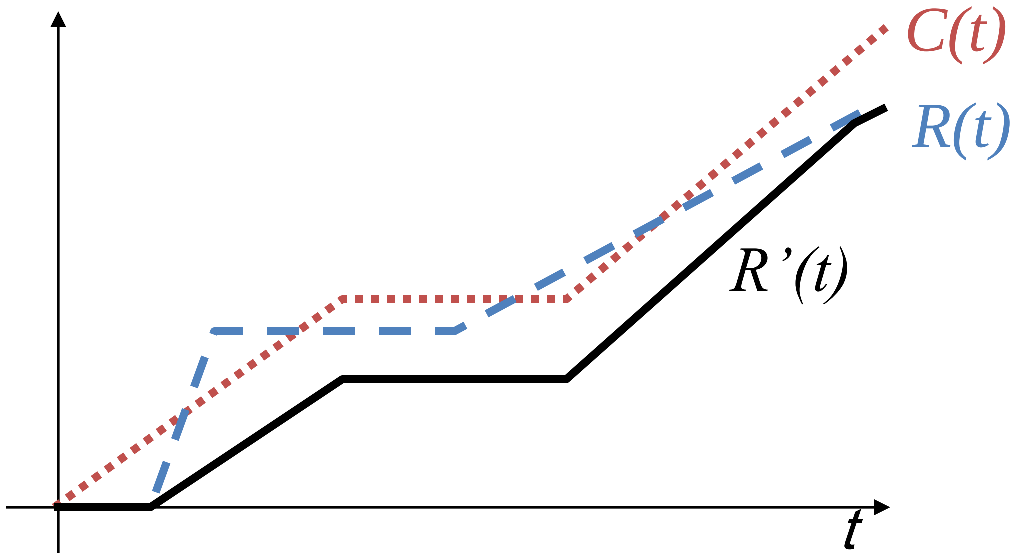
$$C(t) = C'(t) + R'(t)$$

$$B(t) = R(t) - R'(t)$$

Conservation Laws

$$R'(t) = \inf_{0 \le u \le t} \{R(u) + C(t) - C(u)\}$$

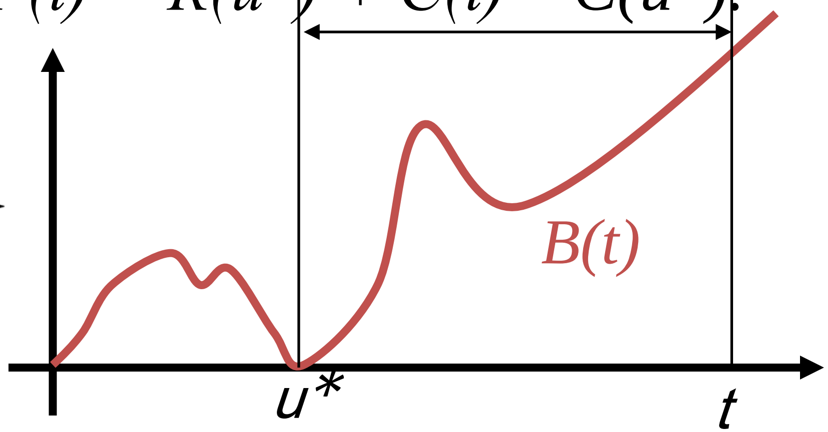




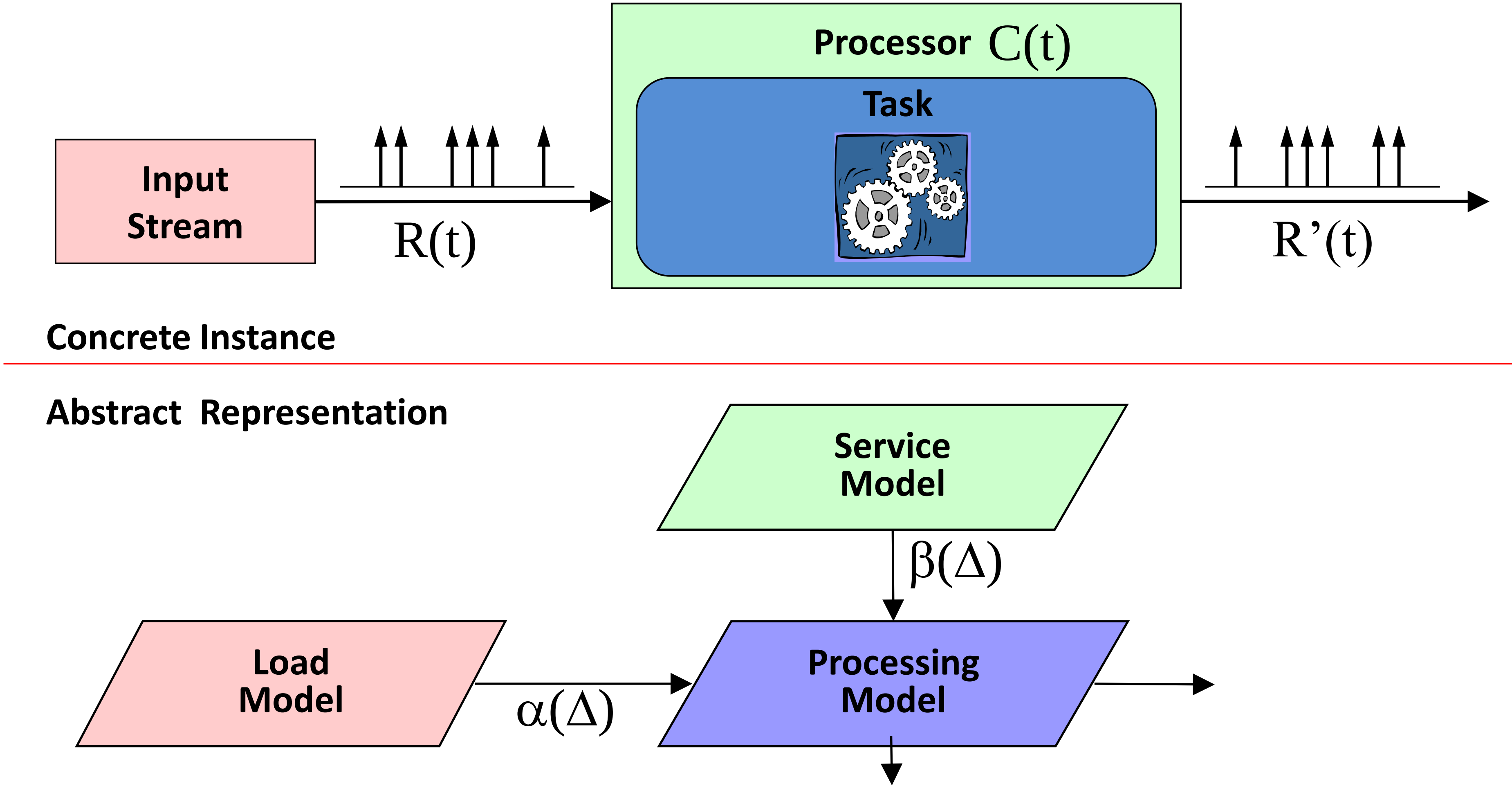
Greedy Processing

- For all times $u \leq t$ we have $R'(u) \leq R(u)$ (conservation law).
- We also have $R'(t) \leq R'(u)+C(t)–C(u)$ as the output can not be larger than the available resources.
- Combining both statements yields $R'(t) \leq R(u) + C(t) – C(u)$.
- Let us suppose that u^* is the last time before *t* with an empty buffer. We have $R(u^*) = R'(u^*)$ at u^* and also $R'(t) = R'(u^*) + C(t) – C(u^*)$ as all available resources are used to produce output. Therefore, $R'(t) = R(u^*) + C(t) – C(u^*)$.
- As a result, we obtain

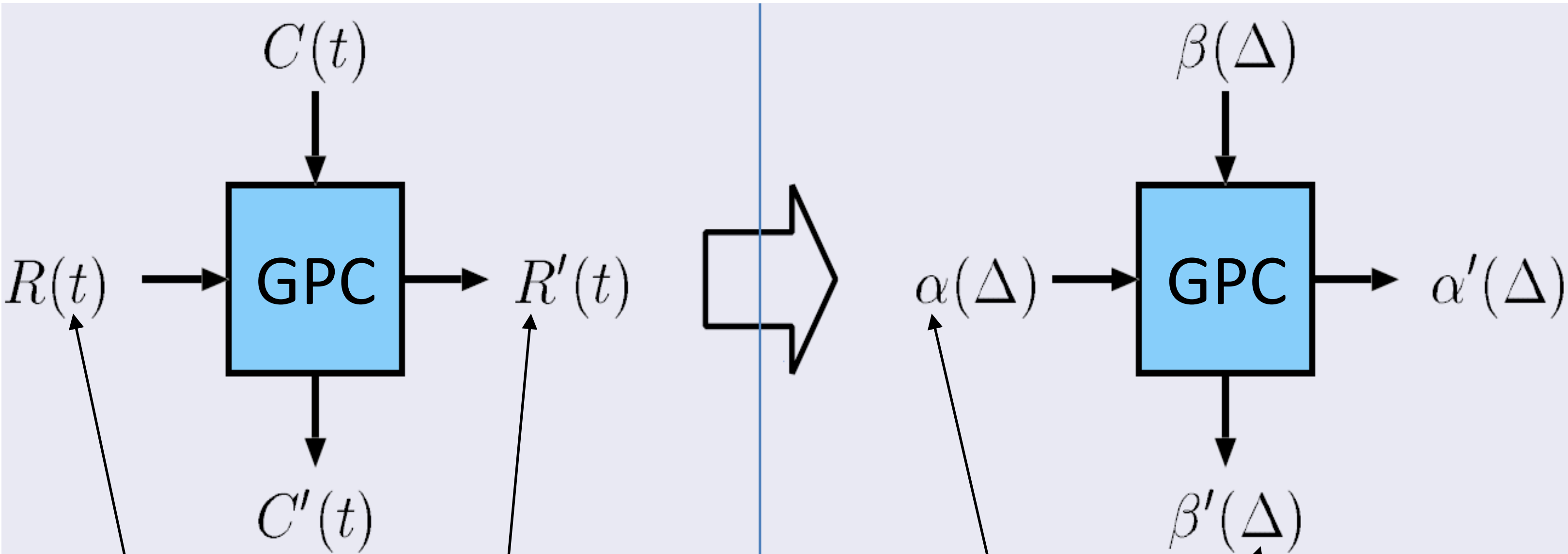
$$R'(t) = \inf_{0 \leq u \leq t} \{R(u) + C(t) - C(u)\}$$



Abstract Models for Performance Analysis



Abstraction



time domain
cumulative functions

time-interval domain
variability curves

Some Definitions and Relations

- $f \otimes g$ is called ***min-plus convolution***

$$(f \otimes g)(t) = \inf_{0 \le u \le t} \{f(t-u) + g(u)\}$$

- $f \oslash g$ is called ***min-plus de-convolution***

$$(f \oslash g)(t) = \sup_{u \ge 0} \{f(t+u) - g(u)\}$$

- For ***max-plus convolution and de-convolution***:

$$(f \overline{\otimes} g)(t) = \sup_{0 \le u \le t} \{f(t-u) + g(u)\}$$

$$(f \overline{\oslash} g)(t) = \inf_{u \ge 0} \{f(t+u) - g(u)\}$$

- Relation between ***convolution and deconvolution***

$$f \le g \otimes h \Leftrightarrow f \oslash h \le g$$

Arrival and Service Curve

- The arrival and service curves provide bounds on event and resource functions as follows:

$$\alpha^l(t-s) \leq R(t) - R(s) \leq \alpha^u(t-s) \quad \forall s \leq t$$

$$\beta^l(t-s) \leq C(t) - C(s) \leq \beta^u(t-s) \quad \forall s \leq t$$

- We can determine valid variability curves from cumulative functions as follows:

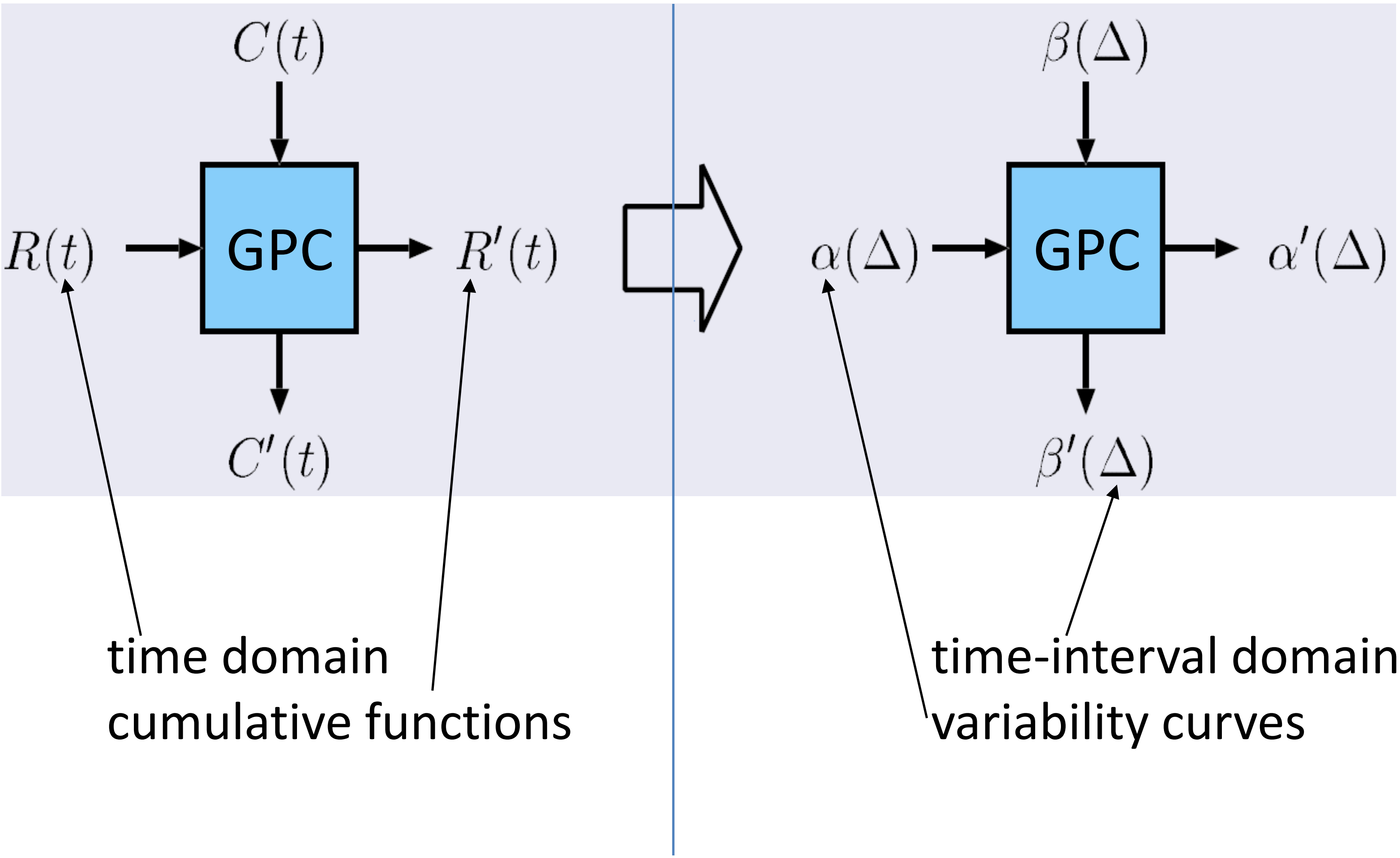
$$\alpha^u = R \oslash R; \quad \alpha^l = R \overline{\oslash} R; \quad \beta^u = C \oslash C; \quad \beta^l = C \overline{\oslash} C$$

- One proof:

$$\alpha^u = R \oslash R \Rightarrow \alpha^u(\Delta) = \sup_{u \geq 0} \{R(\Delta + u) - R(u)\} \Rightarrow$$

$$\alpha^u(\Delta) = \sup_{s \geq 0} \{R(\Delta + s) - R(s)\} \Rightarrow \alpha^u(t-s) \geq R(t) - R(s) \ \ \forall t \geq s$$

Abstraction



The Most Simple Relations

- The *output stream* of a component satisfies:

$$R'(t) \geq (R \otimes \beta^l)(t)$$

- The *output upper arrival curve* of a component satisfies:

$$\alpha^{u\prime} = (\alpha^u \oslash \beta^l)$$

- The *remaining lower service curve* of a component satisfies:

$$\beta^{l\prime}(\Delta) = \sup_{0 \leq \lambda \leq \Delta} (\beta^l(\lambda) - \alpha^u(\lambda))$$

Two Sample Proofs

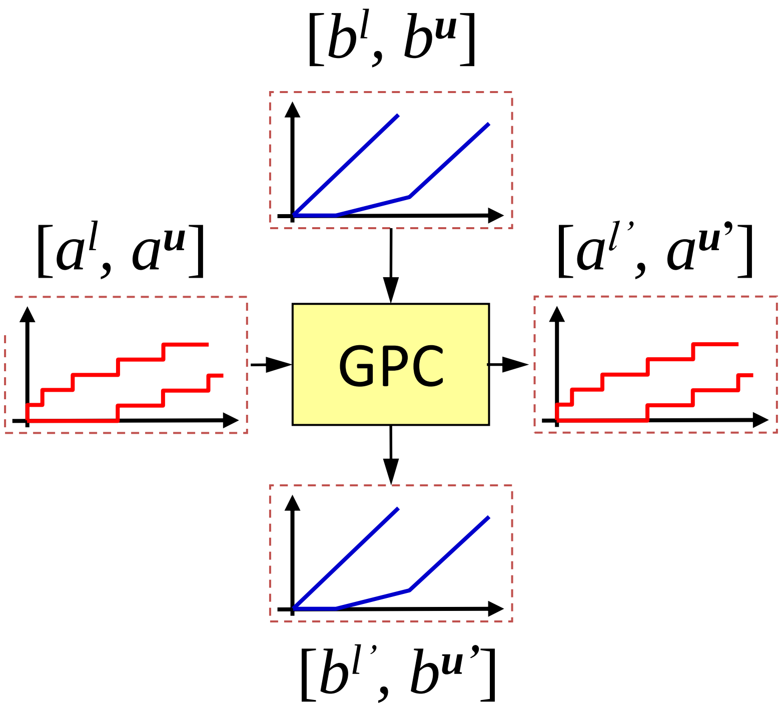
$$R'(t) = \inf_{0 \le u \le t} \{R(u) + C(t) - C(u)\}$$
$$\ge \inf_{0 \le u \le t} \{R(u) + \beta^l(t-u)\}$$
$$= (R \otimes \beta^l)(t)$$

$$C'(t) - C'(s) = \sup_{0 \le a \le t} \{C(a) - R(a)\} - \sup_{0 \le b \le s} \{C(b) - R(b)\} =$$
$$= \inf_{0 \le b \le s} \{ \sup_{0 \le a \le t} \{(C(a) - C(b)) - (R(a) - R(b))\}\}$$
$$= \inf_{0 \le b \le s} \{ \sup_{0 \le a-b \le t-b} \{(C(a) - C(b)) - (R(a) - R(b))\}\}$$
$$\ge \inf_{0 \le b \le s} \{ \sup_{0 \le \lambda \le t-b} \{\beta^l(\lambda) - \alpha^u(\lambda)\}\} \ge \sup_{0 \le \lambda \le t-s} \{\beta^l(\lambda) - \alpha^u(\lambda)\}$$

Tighter Bounds

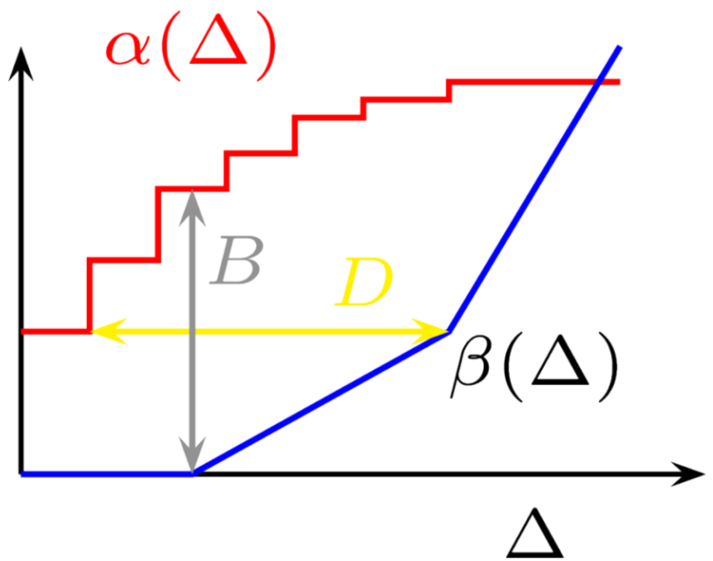
- The greedy processing component transforms the variability curves as follows:

$$\alpha^{u\prime} = [(\alpha^u \otimes \beta^u) \oslash \beta^l] \wedge \beta^u$$
$$\alpha^{l\prime} = [(\alpha^l \oslash \beta^u) \otimes \beta^l] \wedge \beta^l$$
$$\beta^{u\prime} = (\beta^u - \alpha^l)\overline{\oslash}0$$
$$\beta^{l\prime} = (\beta^l - \alpha^u)\overline{\otimes}0$$

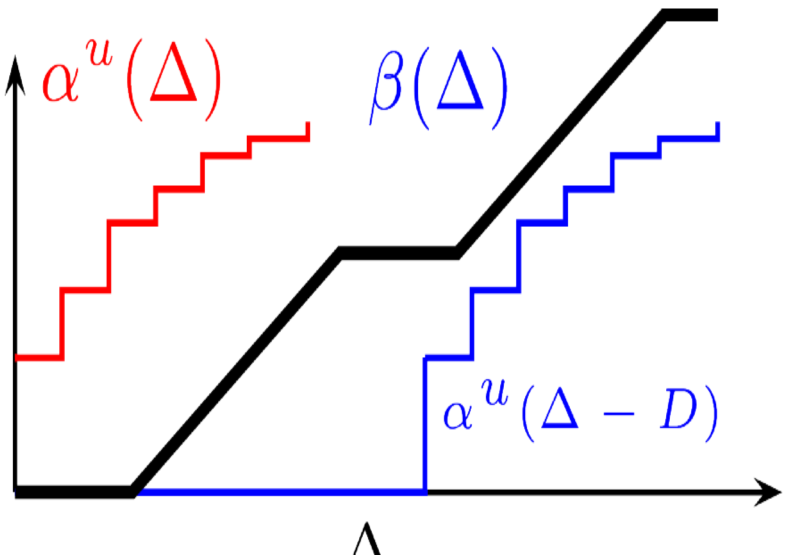


Without proof ... .

Performance Analysis Based on Arrival Curves



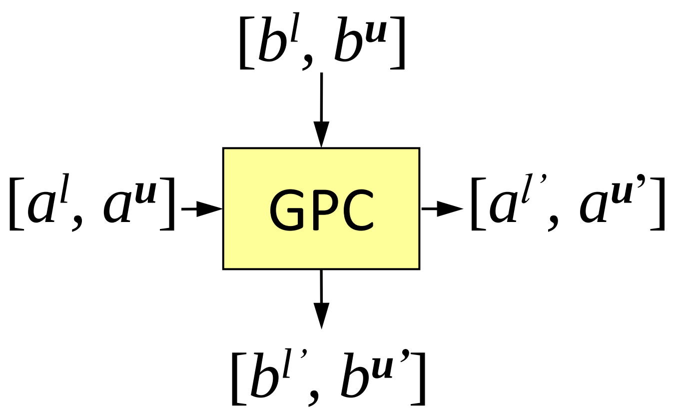
Backlog & Delay

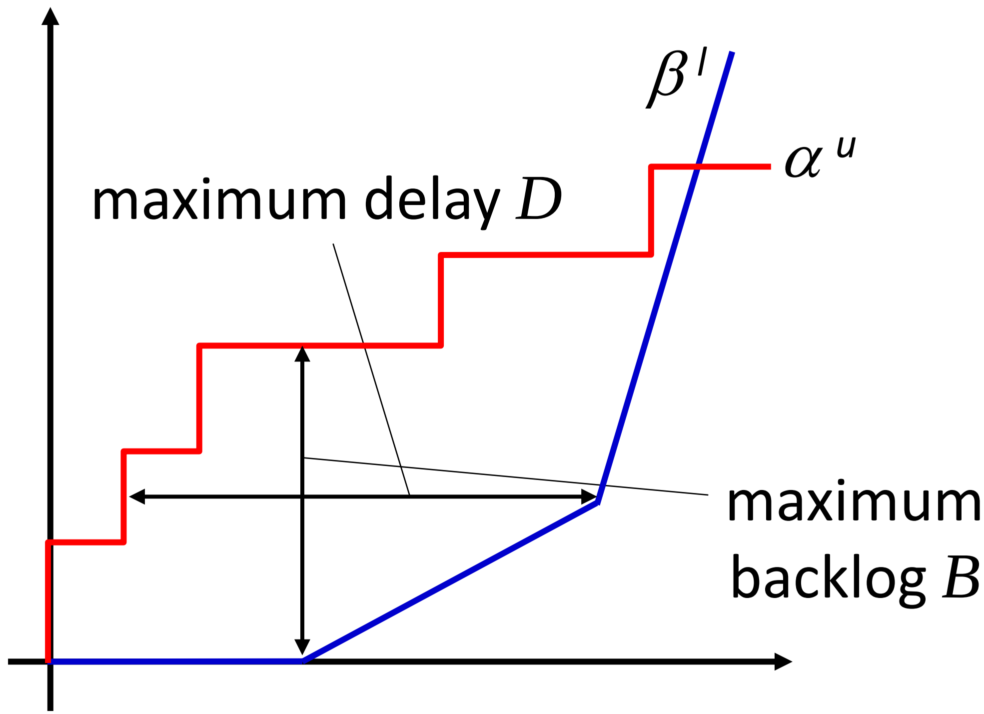


Schedulability

- Close-form solution
- Modular and composition

Delay and Backlog





$$B = \sup_{t \geq 0} \{R(t) - R'(t)\} \leq \sup_{\lambda \geq 0} \left\{\alpha^u(\lambda) - \beta^l(\lambda)\right\}$$

$$D = \sup_{t \geq 0} \{\inf \{\tau \geq 0 : R(t) \leq R'(t + \tau))\}\}$$
$$= \sup_{\Delta \geq 0} \left\{\inf \left\{\tau \geq 0 : \alpha^u(\Delta) \leq \beta^l(\Delta + \tau)\right\}\right\}$$

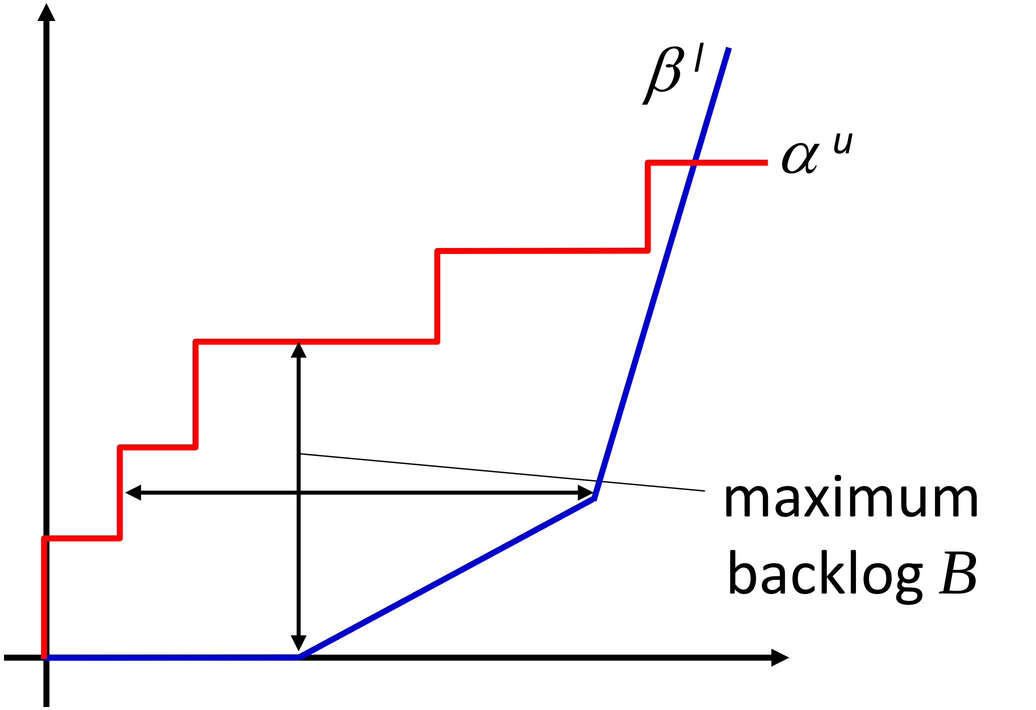
Proof of Backlog Bound

$$B(t) = R(t) - R'(t) = R(t) - \inf_{0 \le u \le t}\{R(u) + C(t) - C(u)\}$$

$$= \sup_{0 \le u \le t}\{(R(t) - R(u)) - (C(t) - C(u))\}$$

$$\le \sup_{0 \le u \le t}\{\alpha^u(t-u) - \beta^l(t-u)\}$$

$$\le \sup_{0 \le \lambda}\{\alpha^u(\lambda) - \beta^l(\lambda)\}$$



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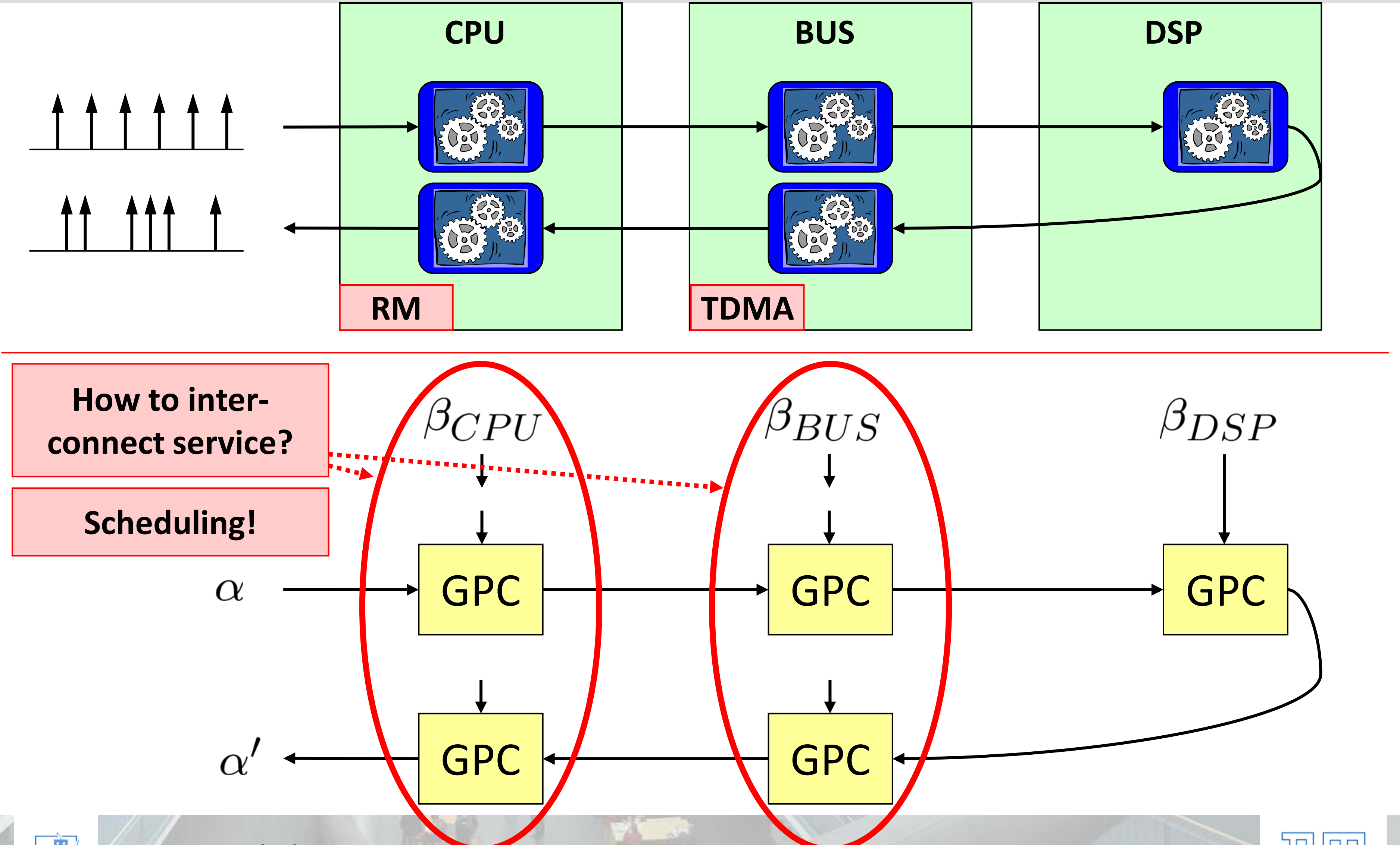
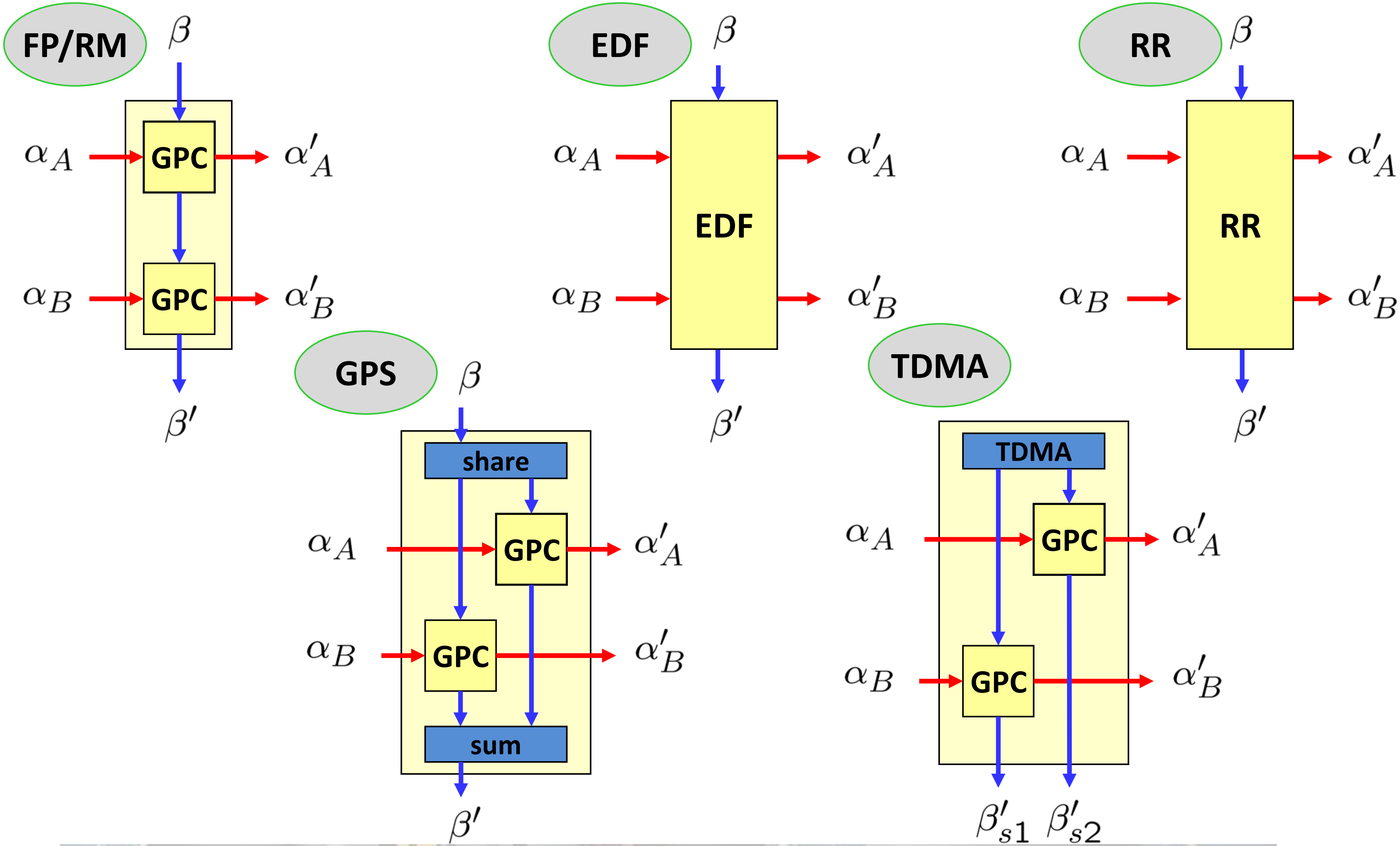
System Composition

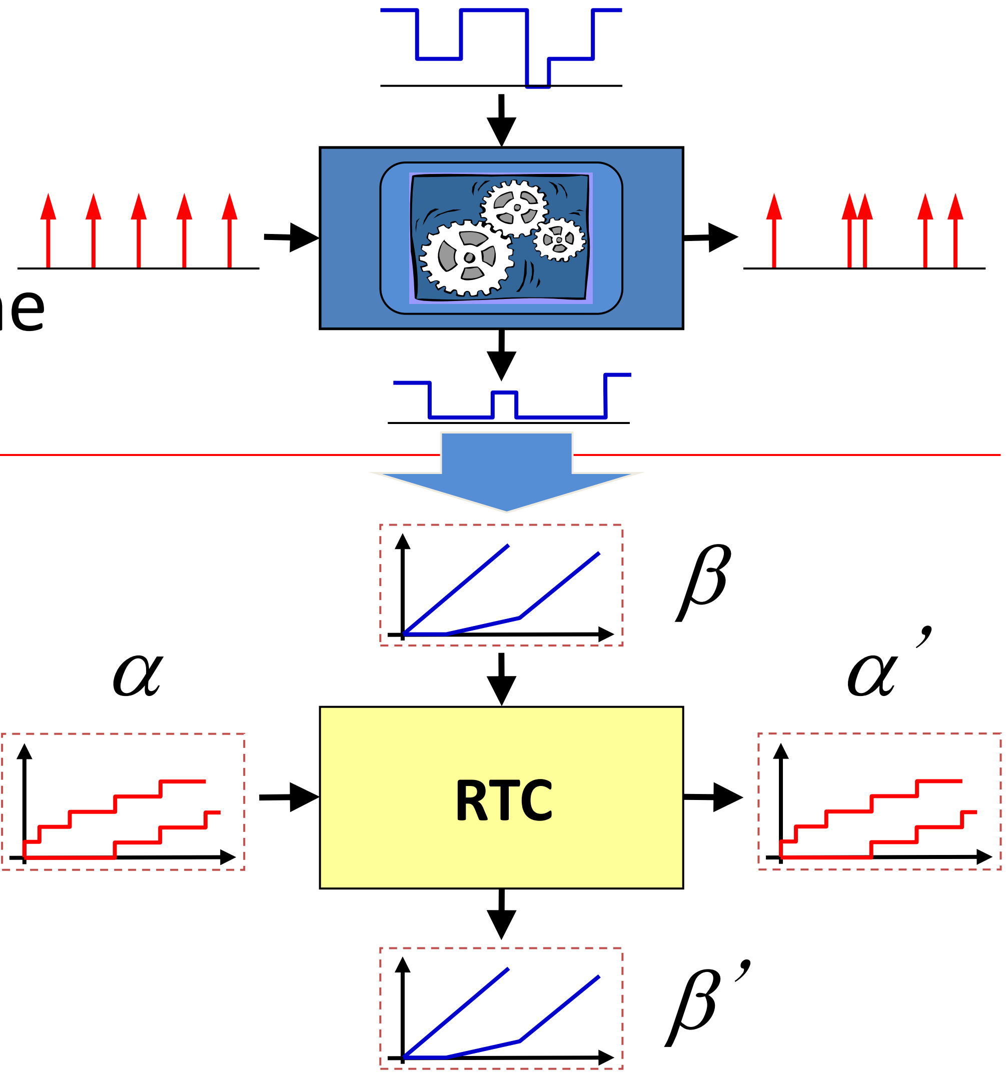


Scheduling and Arbitration



Extending the Framework

- New HW behavior
- New SW behavior
- New scheduling scheme
- ...

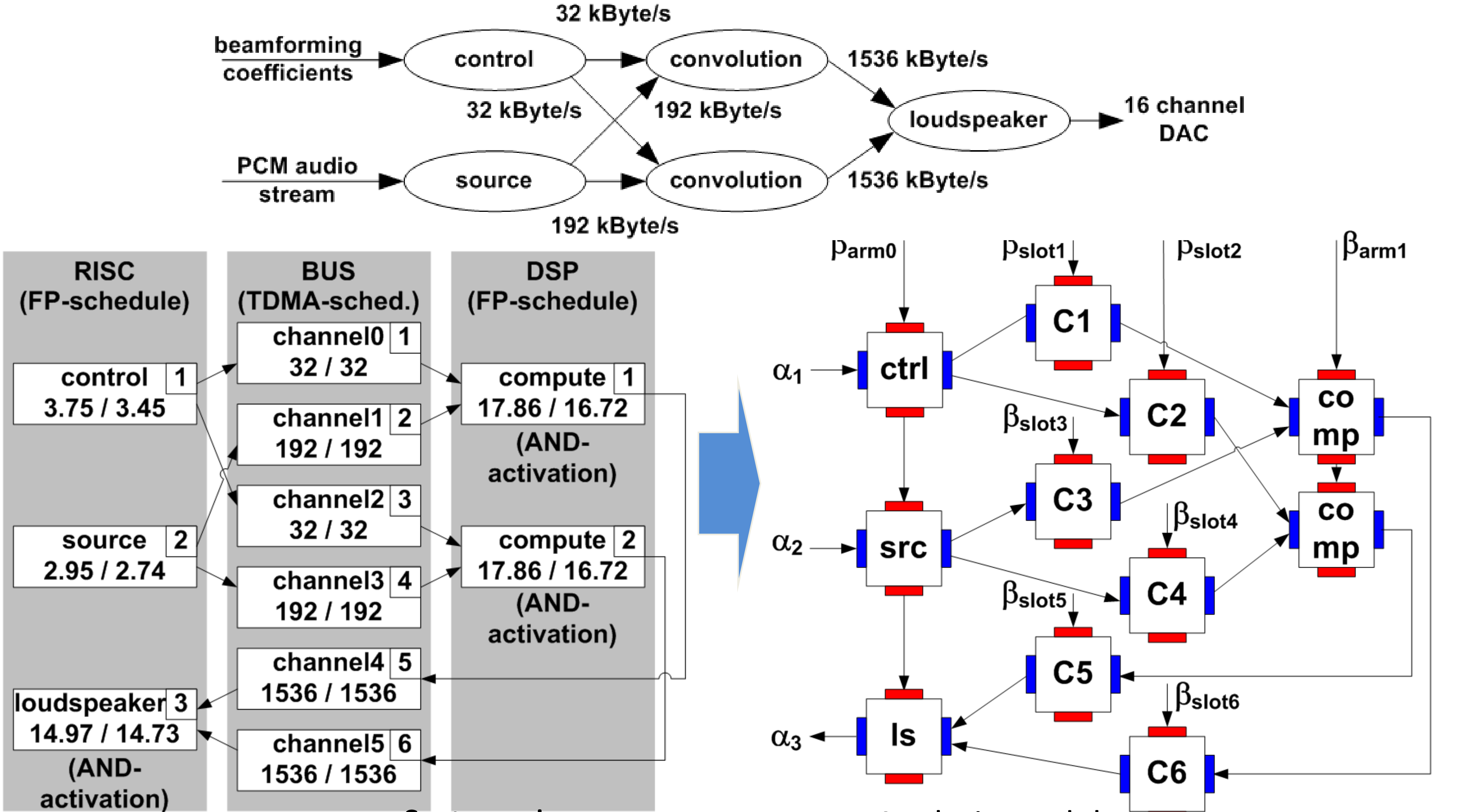


- Find new relations:

$$\begin{aligned} \alpha'(\Delta) &= f_\alpha(\alpha, \beta) \\ \beta'(\Delta) &= f_\beta(\alpha, \beta) \end{aligned}$$

This is the hard part…!

Wave Field Synthesis Example



System view

Analysis model

Wave Field Synthesis Example

- Duration of Analysis Model Generation and Calibration

Step		Duration P–C	Duration MJPEG	Duration WFS
model calibration (one-time effort)	functional simulation generation	22 s	42 s	35 s
	functional simulation	0.2 s	3.6 s	2.4 s
	synthesis (generation of binary)	2 s	4 s	3 s
	simulation on MPARM	23 s	13550 s	740 s
	log-file analysis and back-annotation	1 s	12 s	3 s
model generation		1 s	1 s	1 s
performance analysis based on generated model		0.2 s	2.5 s	1.4 s

Note: Measured on a 1.86 GHz Intel Pentium Mobile machine with 1 GB of RAM.