Vorlesung

Grundlagen der Künstlichen Intelligenz

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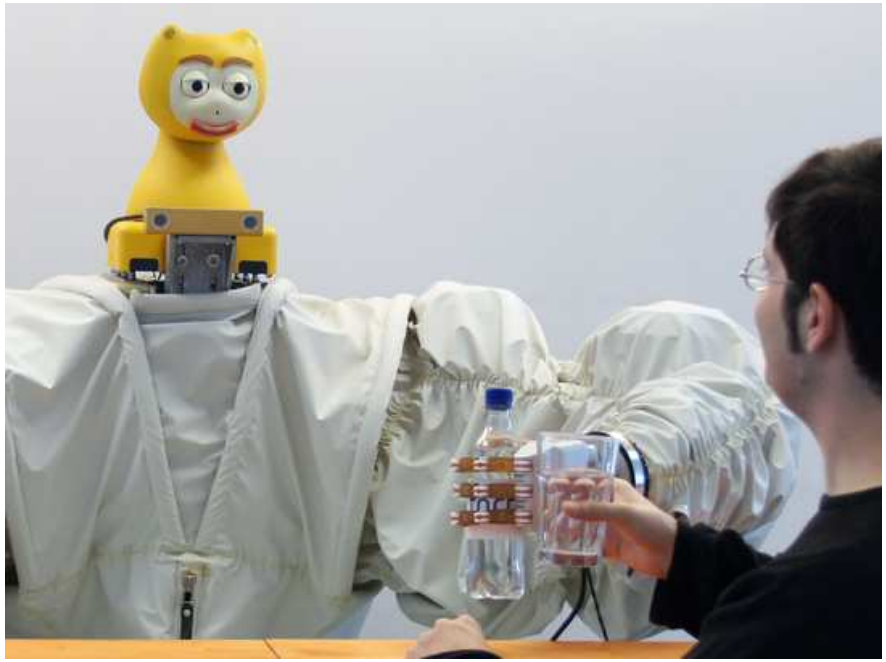

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Chapter 7 (3rd ed., cont‘d)

Logics

From last lecture we know

- Wumpus world
- Propositional Logic with syntax and semantics
- Truth-table approach for proofs

Inference by enumeration

Depth-first enumeration of all models is sound and complete

```
function TT-ENTAILS?(KB, α) returns true or false
    symbols ← a list of the proposition symbols in KB and α
    return TT-CHECK-ALL(KB, α, symbols, [])

function TT-CHECK-ALL(KB, α, symbols, model) returns true or false
    if EMPTY?(symbols) then
        if PL-TRUE?(KB, model) then return PL-TRUE?(α, model)
        else return true
    else do
        P ← FIRST(symbols); rest ← REST(symbols)
        return TT-CHECK-ALL(KB, α, rest, EXTEND(P, true, model)) and
               TT-CHECK-ALL(KB, α, rest, EXTEND(P, false, model))
```

For n symbols, time complexity is $O(2^n)$,

space complexity is $O(n)$

Logical equivalence

- Two sentences are logically equivalent if and only if they are true in same models: $\alpha \equiv ß$ **iff** $\alpha \models \beta$ and $\beta \models \alpha$

$$
\begin{aligned}
(\alpha \wedge \beta) &\equiv (\beta \wedge \alpha) && \text{commutativity of } \wedge \\
(\alpha \vee \beta) &\equiv (\beta \vee \alpha) && \text{commutativity of } \vee \\
((\alpha \wedge \beta) \wedge \gamma) &\equiv (\alpha \wedge (\beta \wedge \gamma)) && \text{associativity of } \wedge \\
((\alpha \vee \beta) \vee \gamma) &\equiv (\alpha \vee (\beta \vee \gamma)) && \text{associativity of } \vee \\
\neg(\neg\alpha) &\equiv \alpha && \text{double-negation elimination} \\
(\alpha \Rightarrow \beta) &\equiv (\neg\beta \Rightarrow \neg\alpha) && \text{contraposition} \\
(\alpha \Rightarrow \beta) &\equiv (\neg\alpha \vee \beta) && \text{implication elimination} \\
(\alpha \Leftrightarrow \beta) &\equiv ((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)) && \text{biconditional elimination} \\
\neg(\alpha \wedge \beta) &\equiv (\neg\alpha \vee \neg\beta) && \text{de Morgan} \\
\neg(\alpha \vee \beta) &\equiv (\neg\alpha \wedge \neg\beta) && \text{de Morgan} \\
(\alpha \wedge (\beta \vee \gamma)) &\equiv ((\alpha \wedge \beta) \vee (\alpha \wedge \gamma)) && \text{distributivity of } \wedge \text{ over } \vee \\
(\alpha \vee (\beta \wedge \gamma)) &\equiv ((\alpha \vee \beta) \wedge (\alpha \vee \gamma)) && \text{distributivity of } \vee \text{ over } \wedge
\end{aligned}
$$

Terminology

- Countable alphabet Σ of consisting of A, B, C, …
- Atom: atomic sentence, i.e. a symbol from the alphabet
- Literal: (possibly negated) atomic sentence, e.g. A, $\neg$B
- Clause: Disjunction of literals, e.g. (A $\vee$ $\neg$B $\vee$ C)

Validity and satisfiability

- A sentence is valid (tautology) if it is true in all models,
  e.g., *True*, $A \vee \neg A$, $A \Rightarrow A$, $(A \wedge (A \Rightarrow B)) \Rightarrow B$

- A sentence is satisfiable if it is true in some model
  e.g., $A \vee B$, C

- A sentence is unsatisfiable if it is true in no models
  e.g., $A \wedge \neg A$

- Validity is connected to inference via the Deduction Theorem:
  $KB \models \alpha$ if and only if $(KB \Rightarrow \alpha)$ is valid

- Satisfiability is connected to inference via:
  $KB \models \alpha$ if and only if $(KB \wedge \neg \alpha)$ is unsatisfiable

Proof methods

Proof methods divide into (roughly) two kinds:

- Application of inference rules
  - Legitimate (sound) generation of new sentences from existing ones
  - Proof = a sequence of inference rule applications
  - Algorithms can use inference rules as operators in a standard search algorithm
  - Typically require transformation of sentences into a normal form

- Model checking
  - truth table enumeration (always exponential in *n*)
  - improved backtracking, e.g., Davis-Putnam-Logemann-Loveland (DPLL) algorithm
  - heuristic search in model space (sound but incomplete)
    e.g., min-conflicts-like hill-climbing algorithms

Normal forms

Conjunctive normal form (CNF):

The sentence consists of a conjunction of disjunctions of literals $l_{i,j}$, i.e. it has the following form:

$$\bigwedge_{i=1..n}\left(\bigvee_{j=1..m} l_{i,j}\right)$$

e.g., (A ∨ B ∨ ¬C) ∧ (¬D ∨ B) ∧ (E ∨ C ∨ F)

Disjunctive normal form (DNF):

The sentence consists of a disjunction of conjunctions of literals $l_{i,j}$, i.e. it has the following form:

$$\bigvee_{i=1..n}\left(\bigwedge_{j=1..m} l_{i,j}\right)$$

e.g., (A ∧ B ∧ ¬C) ∨ (¬D ∧ B) ∨ (E ∧ C ∧ F)

Normal forms - properties

- For all sentences, there is at least one equivalent sentence in CNF and DNF.
- A sentence in DNF is satisfiable **iff** a disjunction is satisfiable.
- A sentence in KNF is valid **iff** every conjunction is valid.

Construction of CNF sentences

1. Eliminate $\Rightarrow$ and $\Leftrightarrow$: $A \Rightarrow B$ ~> $(\neg A \vee B)$
2. Move $\neg$ to the inside: $\neg(A \wedge B)$ ~> $(\neg A \vee \neg B)$
3. Distribute $\vee$ over $\wedge$: $(A \wedge B) \vee C$ ~> $(A \vee C) \wedge (B \vee C)$
4. Simplify: $(A \vee A)$ ~> A

The result is a conjunction of disjunction of literals.

- An analogous procedure transforms any sentence into an equivalent sentence in DNF.
- Sentences can be expanded exponentially during the transformation.

Inference rules

Modus ponens

$$\frac{A \Rightarrow B,\ A}{B}$$

Conjunction Elimination

$$\frac{A_1 \wedge A_2 \wedge A_3 \wedge \ldots \wedge A_n}{A_i}$$

Conjunction Introduction

$$\frac{A_1,\ A_2,\ A_3,\ \ldots,\ A_n}{A_1 \wedge A_2 \wedge A_3 \wedge \ldots \wedge A_n}$$

Disjunction Introduction

$$\frac{A_i}{A_1 \vee A_2 \vee A_3 \vee \ldots \vee A_n}$$

Inference rules - Resolution

Unit resulotion

$$\frac{A \vee B,\ \neg A}{B}$$

Resolution

$$\frac{A \vee B,\ \neg B \vee C}{A \vee C}$$

or, more general

$$\frac{A_1 \vee A_2 \vee \ldots \vee A_n,\quad \neg A_1 \vee B_2 \vee \ldots \vee B_n}{A_2 \vee \ldots \vee A_n \vee B_2 \vee \ldots \vee B_n}$$

Proof by Resolution - Idea

- We now want to study a deduction technique that does not rely on explicitly testing all interpretations.
- Idea: You try to show that a set of sentences is unsatisfiable.
- However: It is required that all sentences are given in CNF.
- But: In most cases, the sentences are close to their CNF (and there is a quick transformation that preserves satisfyability).
- Nevertheless: In the worst case, this deduction technique also requires an exponential amount of time (probably, you cannot avoid this).

Proof by Resolution

To prove that KB |= A , show that (KB $\wedge \neg$A) is unsatisfiable

Use the inference rules including th resolution rules to infer the empty clause $\square$ from (KB $\wedge \neg$A)

E.g. show that in the Wumpus world there is no pit at [1,2],

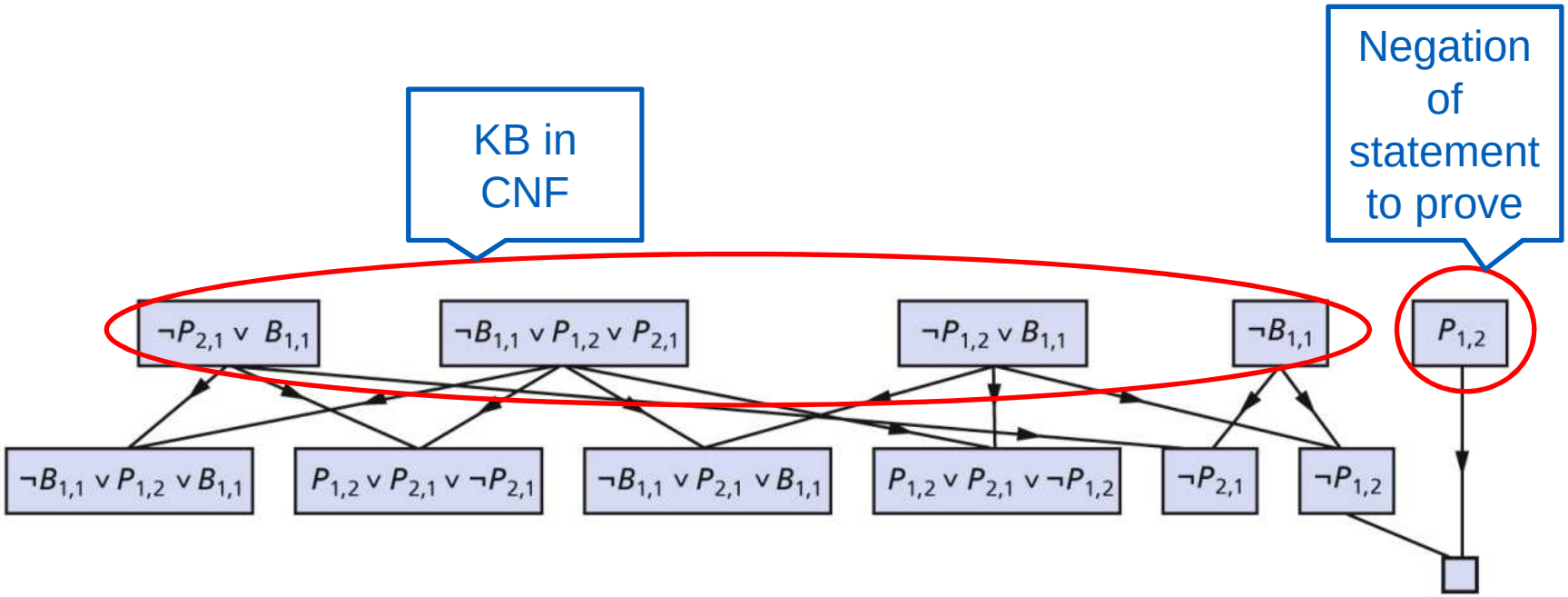
formal: $\neg P_{1,2}$

The Knowledge base is

$KB = (B_{1,1} \Rightarrow (P_{1,2} \vee P_{2,1})) \wedge \neg B_{1,1}$

Proof by Resolution

Add the negated clause you want to show to the KB:



A simple resolution algorithm

function PL-RESOLUTION(KB, α) **returns** *true* or *false*

clauses ← the set of clauses in the CNF representation of $KB \wedge \neg\alpha$
new ← { }
loop do
  for each C_i, C_j **in** *clauses* **do**
    resolvents ← PL-RESOLVE(C_i, C_j)
    if *resolvents* contains the empty clause **then return** *true*
    new ← *new* ∪ *resolvents*
  if *new* ⊆ *clauses* **then return** *false*
  clauses ← *clauses* ∪ *new*

Completeness of the resolution

- Resolution theorem:

If a set of clauses is unsatisfiable, the resolution closure of these clauses contains the empty clause

- i.e., the resolution is complete, KB |= A can be shown by deriving $\square$ from (KB $\wedge \neg$A).

Resticted forms of clauses

- Definite clause: dicjunction of literals with exactly one positive literal, e.g. $(A \vee \neg B \vee \neg C)$
- Horn clause: dicjunction of literals with at most one positive literal, e.g. $(A \vee \neg B \vee \neg C)$, $(\neg B \vee \neg C)$

The resolvent of two Horn clauses is again a Horn clause.

Horn clauses can be read as implications:

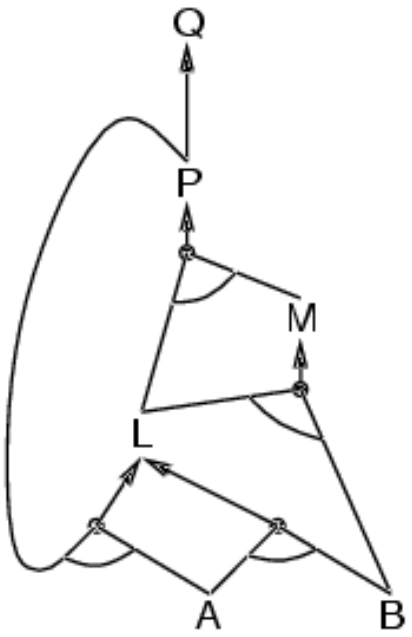
$A \vee \neg B \vee \neg C \quad \equiv \quad (B \wedge C) \Rightarrow A$

Horn clauses can be used for forward- and backward chaining

Forward chaining

- Idea: fire any rule whose premises are satisfied in the *KB*,
  - add its conclusion to the *KB*, until query is found

$$P \Rightarrow Q$$
$$L \wedge M \Rightarrow P$$
$$B \wedge L \Rightarrow M$$
$$A \wedge P \Rightarrow L$$
$$A \wedge B \Rightarrow L$$
$$A$$
$$B$$

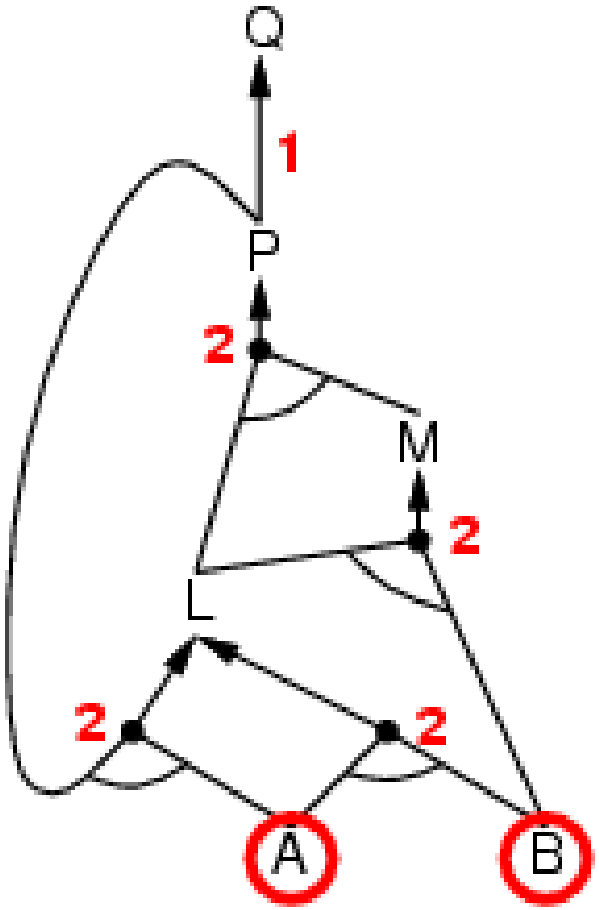


Forward chaining algorithm

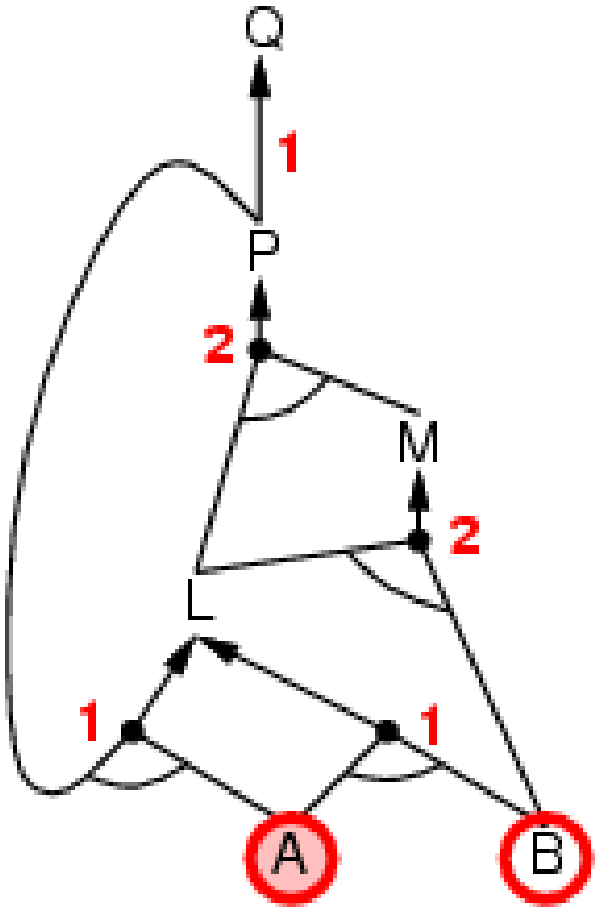
```
function PL-FC-ENTAILS?(KB, q) returns true or false
  local variables: count, a table, indexed by clause, initially the number of premises
                   inferred, a table, indexed by symbol, each entry initially false
                   agenda, a list of symbols, initially the symbols known to be true

  while agenda is not empty do
      p ← POP(agenda)
      unless inferred[p] do
          inferred[p] ← true
          for each Horn clause c in whose premise p appears do
              decrement count[c]
              if count[c] = 0 then do
                  if HEAD[c] = q then return true
                  PUSH(HEAD[c], agenda)
  return false
```

- Forward chaining is sound and complete for Horn KBs

Forward chaining example



Forward chaining example



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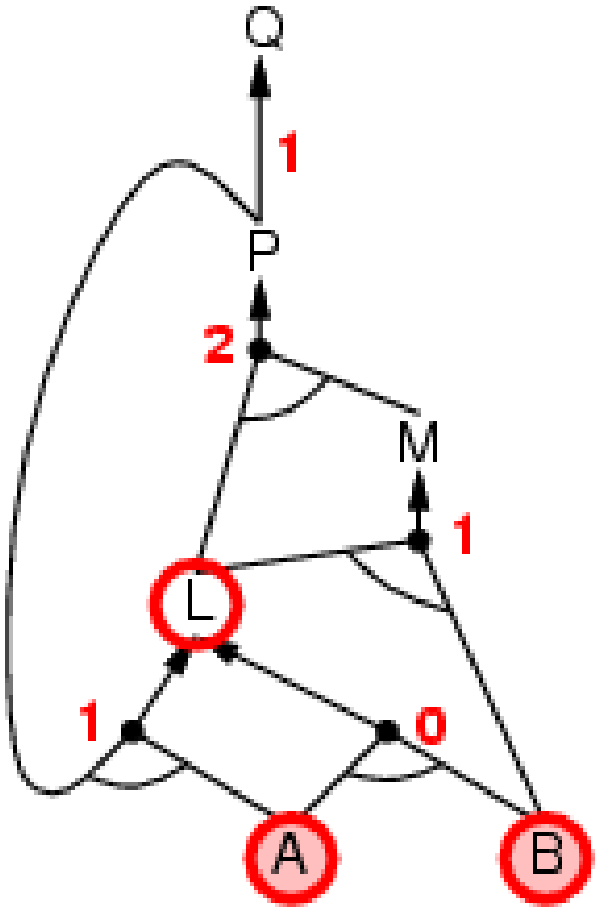

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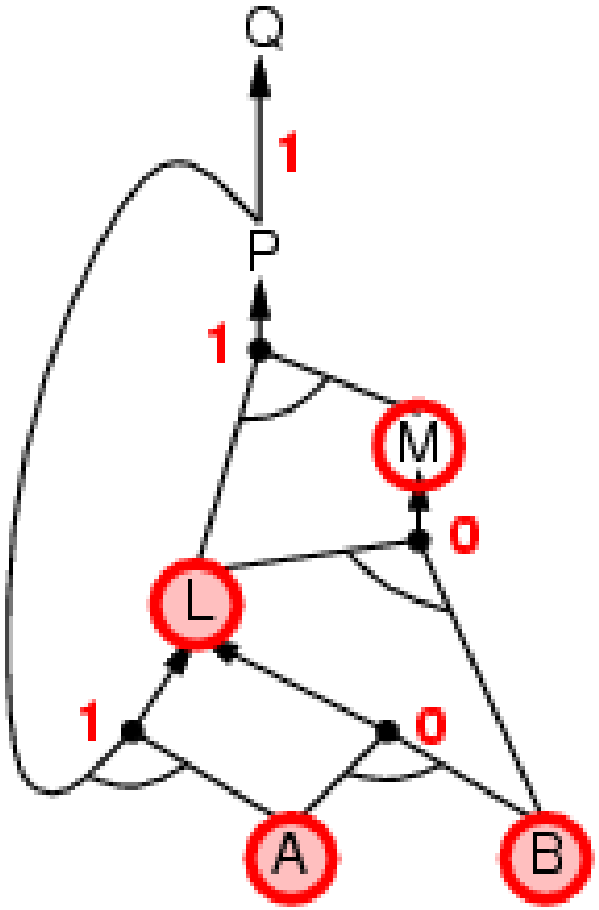

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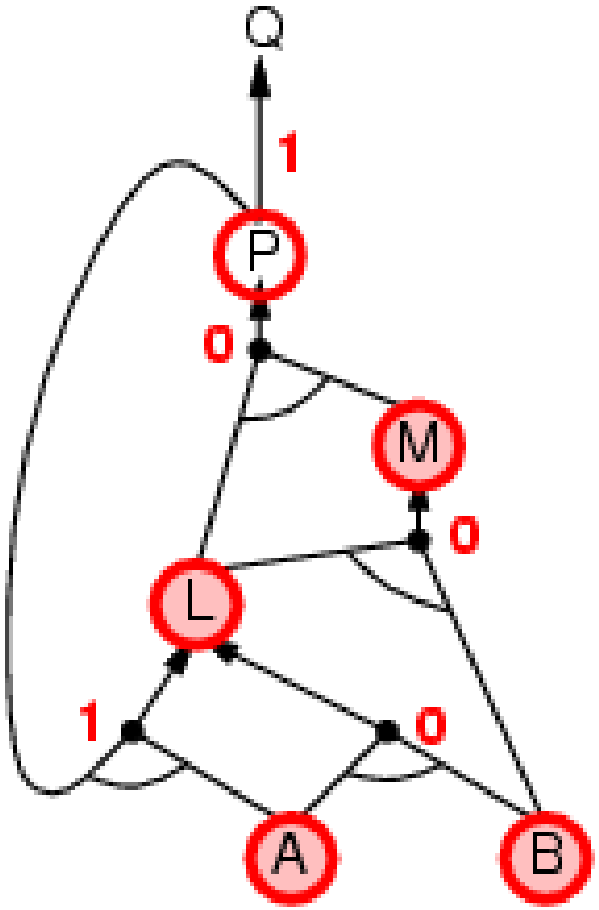

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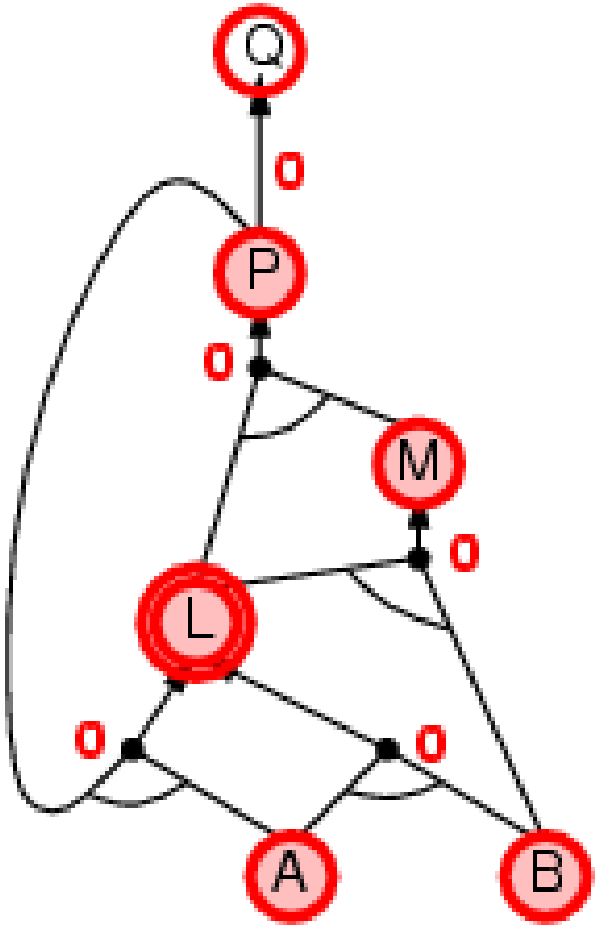

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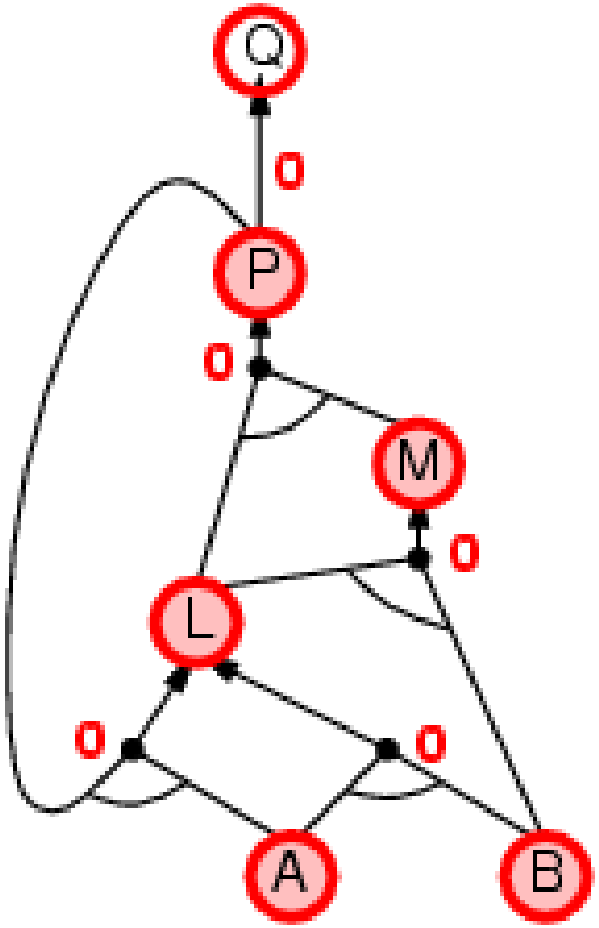

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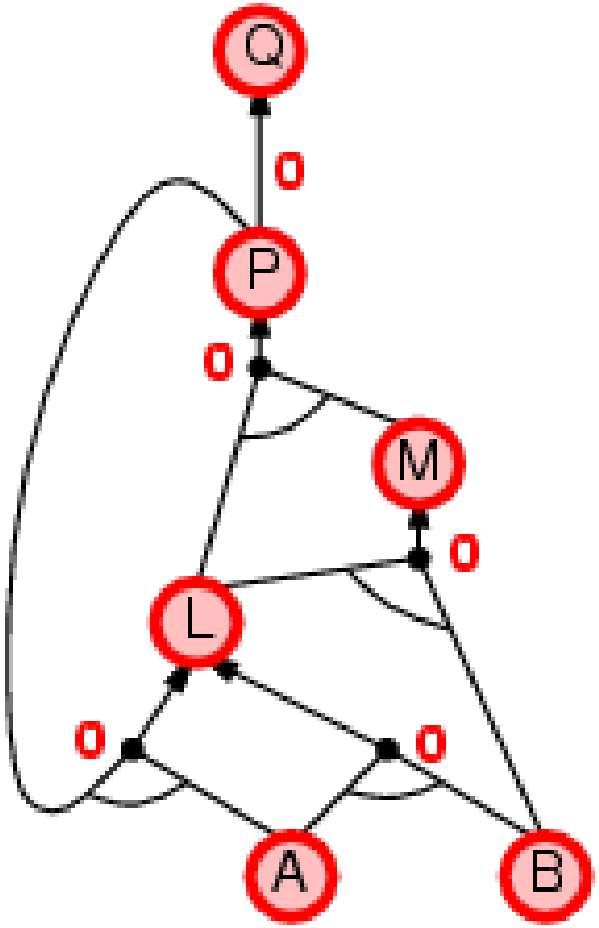


Backward chaining

Idea: work backwards from the query *q*:

to prove *q* by BC,

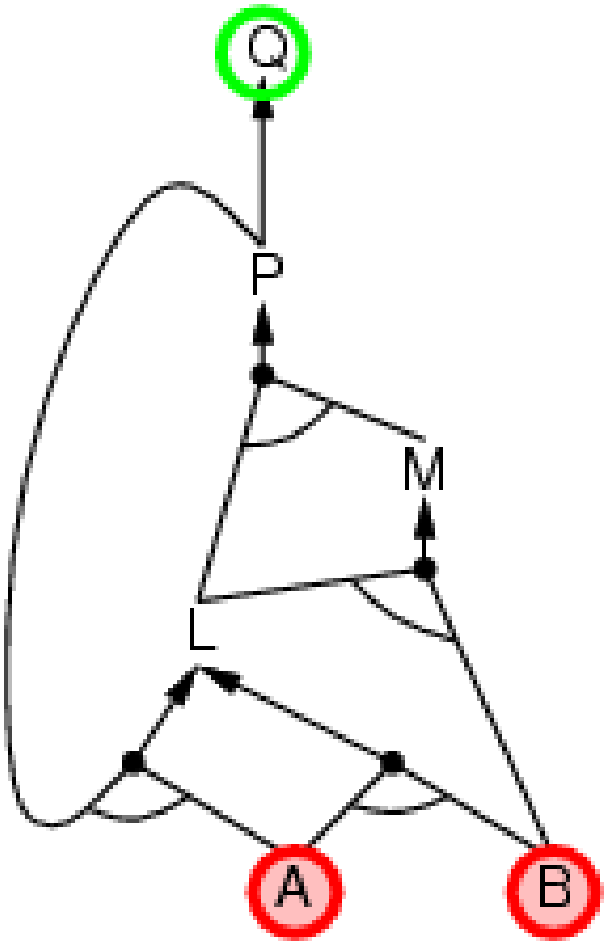
- check if *q* is known already, or
- prove by BC all premises of some rule concluding *q*

Avoid loops: check if new subgoal is already on the goal stack

Avoid repeated work: check if new subgoal

1. has already been proved true, or
2. has already failed

Backward chaining example



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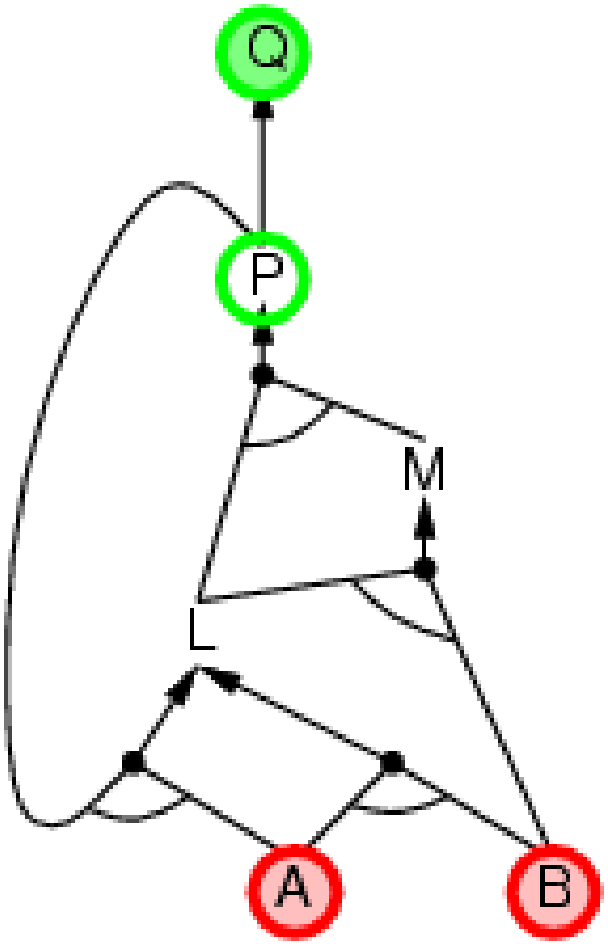

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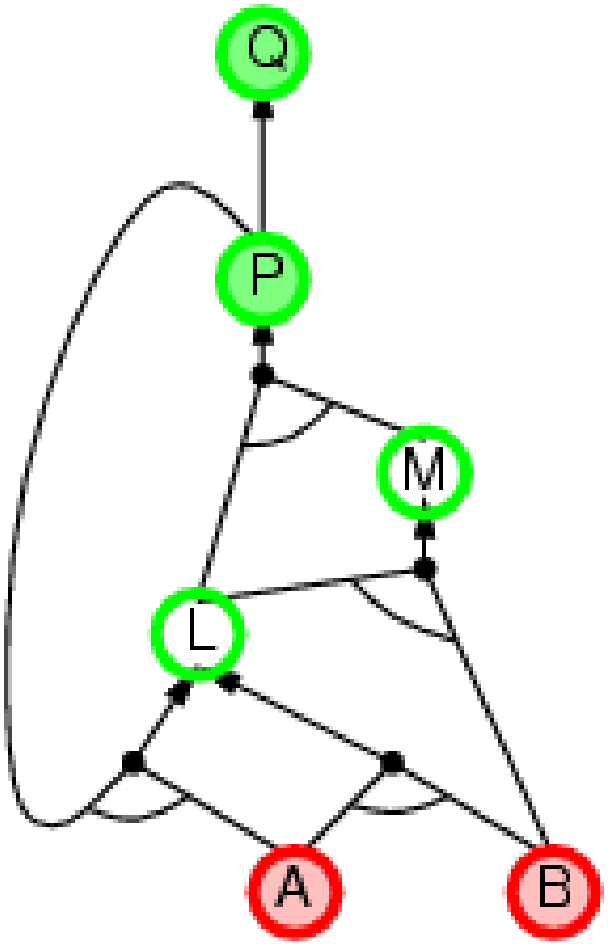

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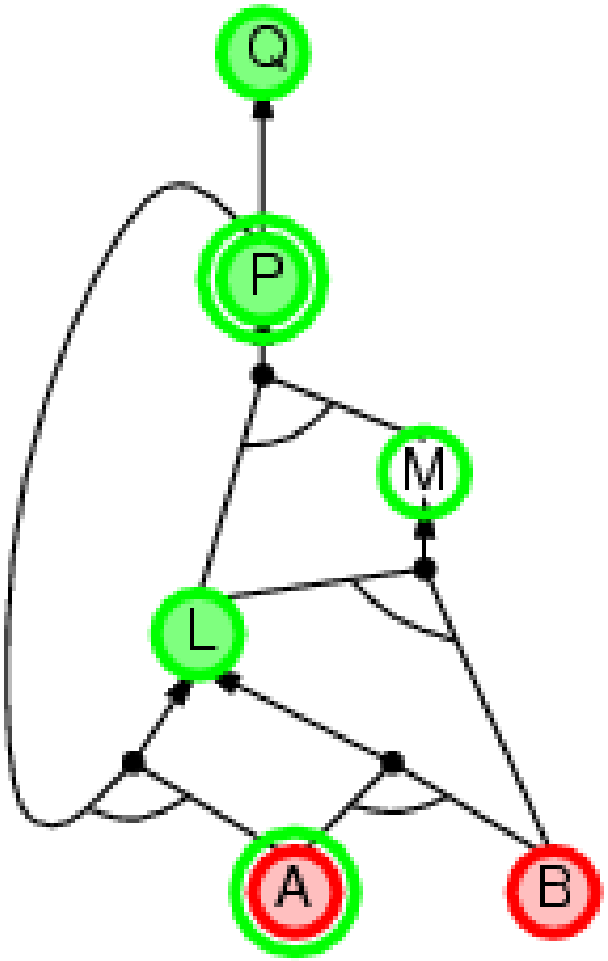

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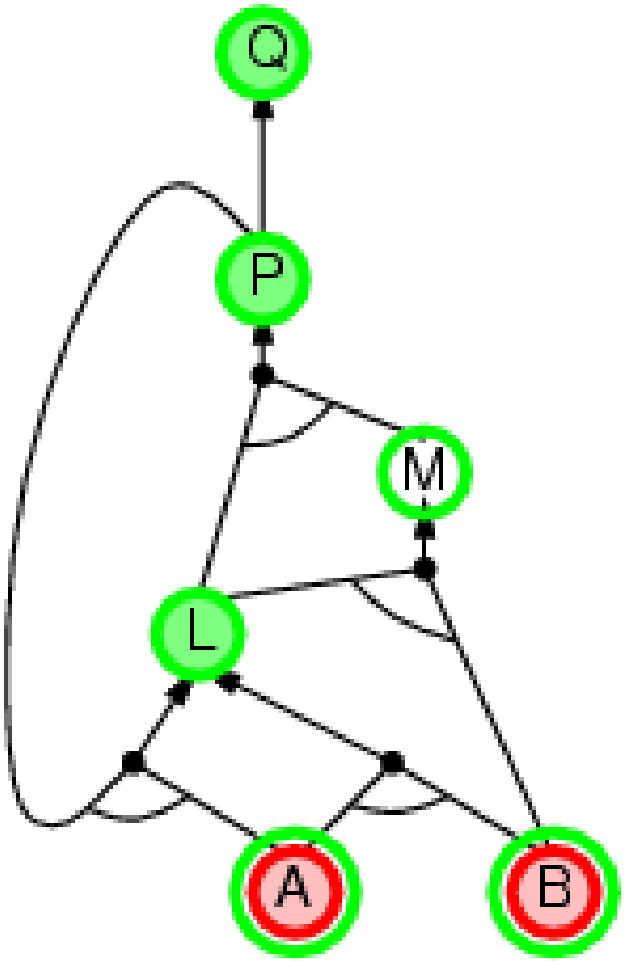

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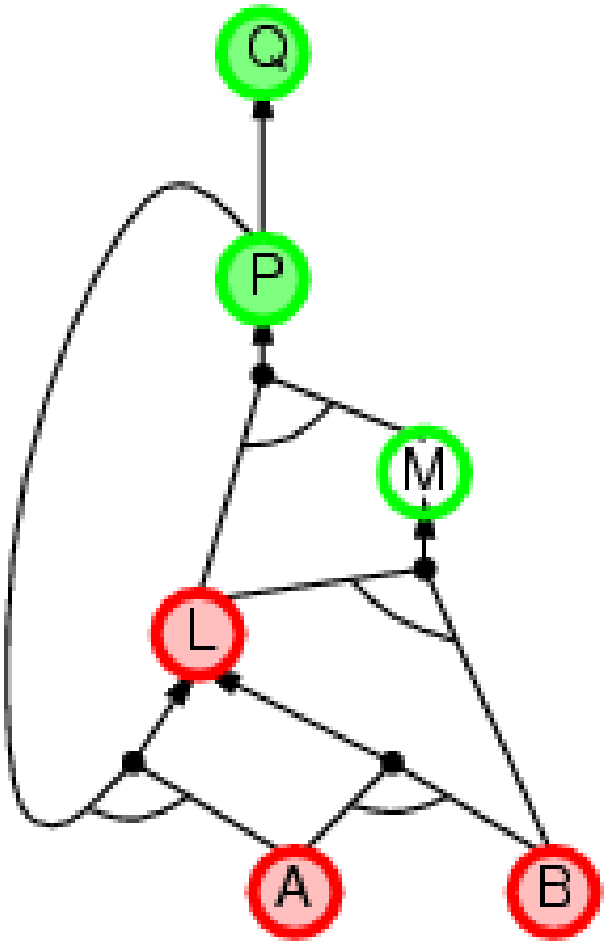

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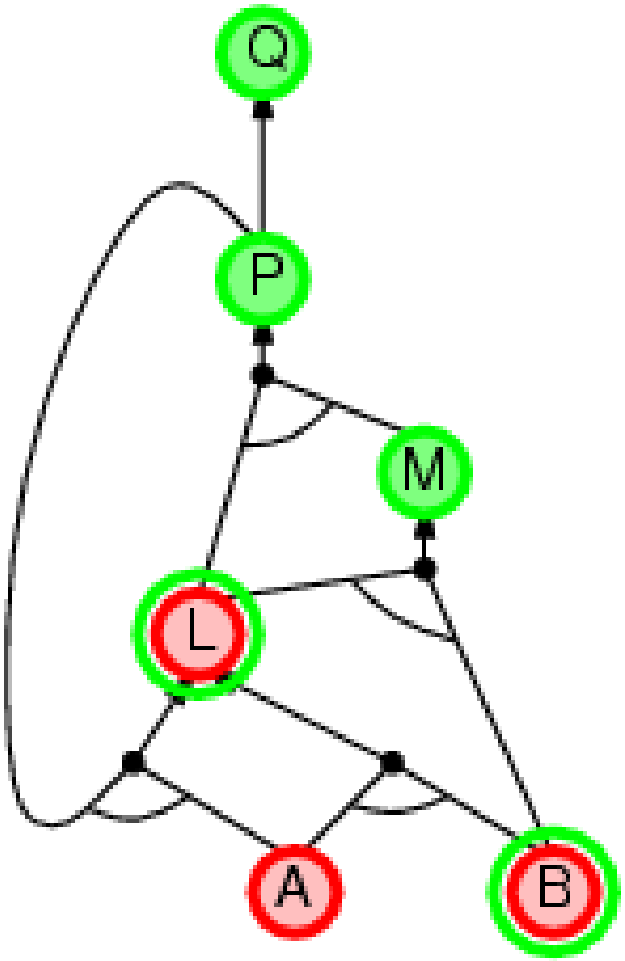

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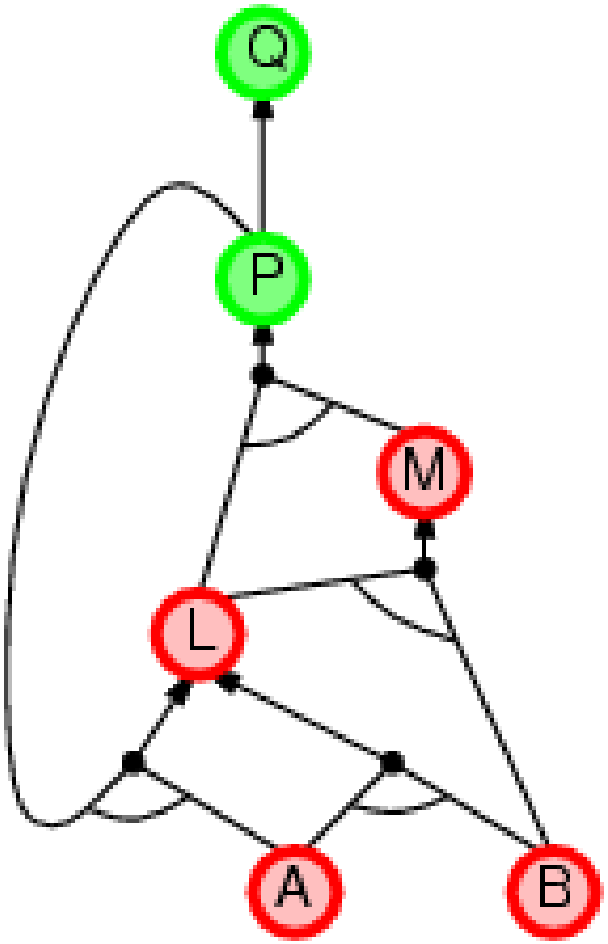

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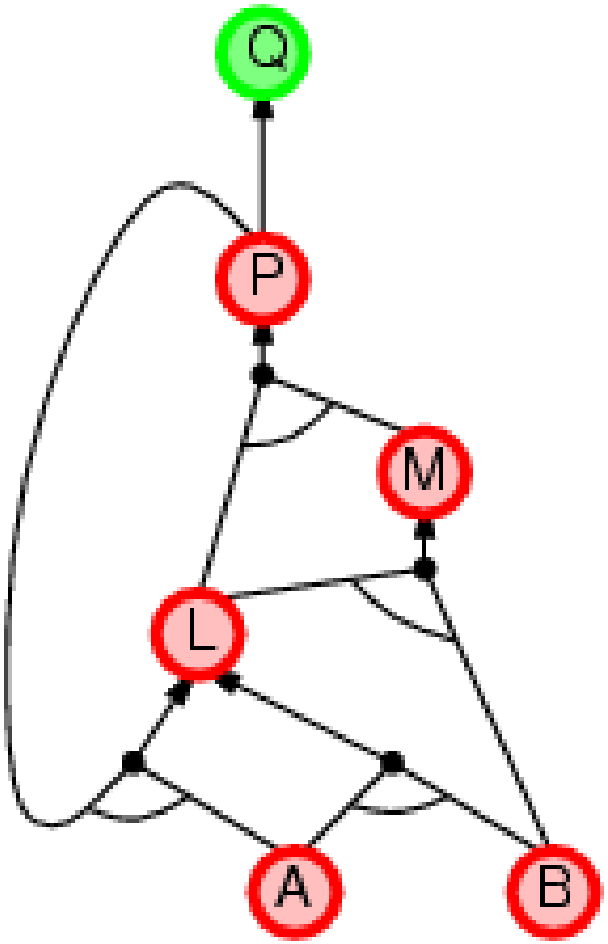

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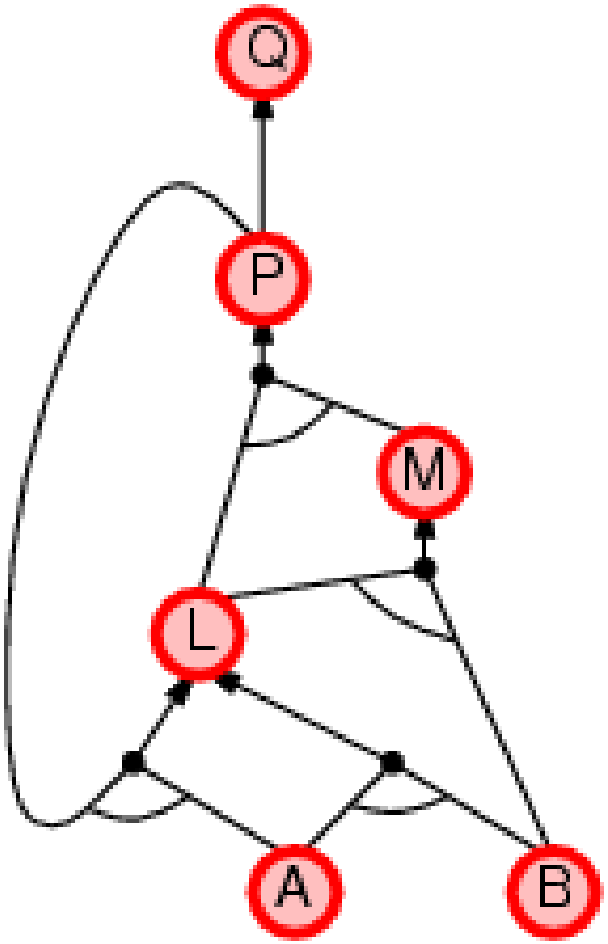

Forward vs. backward chaining

- FC is data-driven, automatic, unconscious processing,
  - e.g., object recognition, routine decisions
- May do lots of work that is irrelevant to the goal
- BC is goal-driven, appropriate for problem-solving,
  - e.g., Where are my keys? How do I get into a PhD program?
- Complexity of BC can be much less than linear in size of KB

Efficient propositional inference

Two families of efficient algorithms for propositional inference:

Complete backtracking search algorithms

- DPLL algorithm (Davis, Putnam, Logemann, Loveland)

- Incomplete local search algorithms
  - `WalkSAT` algorithm

The DPLL algorithm

Determine if an input propositional logic sentence (in CNF) is satisfiable. Improvements over truth table enumeration:

1. Early termination
   A clause is true if any literal is true.
   A sentence is false if any clause is false.

2. Pure symbol heuristic
   Pure symbol: always appears with the same "sign" in all clauses.
   e.g., In the three clauses $(A \vee \neg B)$, $(\neg B \vee \neg C)$, $(C \vee A)$, A and B are pure, C is impure. Make a pure symbol literal true.

3. Unit clause heuristic
   Unit clause: only one literal in the clause
   The only literal in a unit clause must be true.

The DPLL algorithm

function DPLL-SATISFIABLE?(s) **returns** *true* or *false*

inputs: s, a sentence in propositional logic

clauses ← the set of clauses in the CNF representation of s

symbols ← a list of the proposition symbols in s

return DPLL(*clauses*, *symbols*, [])

---

function DPLL(*clauses*, *symbols*, *model*) **returns** *true* or *false*

if every clause in *clauses* is true in *model* **then return** *true*

if some clause in *clauses* is false in *model* **then return** *false*

P, *value* ← FIND-PURE-SYMBOL(*symbols*, *clauses*, *model*)

if P is non-null **then return** DPLL(*clauses*, *symbols*–P, [$P =$ *value*|*model*])

P, *value* ← FIND-UNIT-CLAUSE(*clauses*, *model*)

if P is non-null **then return** DPLL(*clauses*, *symbols*–P, [$P =$ *value*|*model*])

P ← FIRST(*symbols*); *rest* ← REST(*symbols*)

return DPLL(*clauses*, *rest*, [$P =$ *true*|*model*]) **or**
DPLL(*clauses*, *rest*, [$P =$ *false*|*model*])

The `WalkSAT` algorithm

- Incomplete, local search algorithm
- Evaluation function: The min-conflict heuristic of minimizing the number of unsatisfied clauses
- Balance between greediness and randomness

The WalkSAT algorithm

```
function WalkSAT(clauses, p, max-flips) returns a satisfying model or failure
    inputs: clauses, a set of clauses in propositional logic
            p, the probability of choosing to do a "random walk" move
            max-flips, number of flips allowed before giving up

    model ← a random assignment of true/false to the symbols in clauses
    for i = 1 to max-flips do
        if model satisfies clauses then return model
        clause ← a randomly selected clause from clauses that is false in model
        with probability p flip the value in model of a randomly selected symbol
                from clause
        else flip whichever symbol in clause maximizes the number of satisfied clauses
    return failure
```


Summary

- Logical agents apply inference to a knowledge base to derive new information and make decisions

Basic concepts of logic:

- syntax: formal structure of sentences
- semantics: truth of sentences wrt models
- entailment: necessary truth of one sentence given another
- inference: deriving sentences from other sentences
- soundness: derivations produce only entailed sentences
- completeness: derivations can produce all entailed sentences

- Wumpus world requires the ability to represent partial and negated information, reason by cases, etc.
- Resolution is complete for propositional logic
- Forward, backward chaining are linear-time, complete for Horn clauses
- DPLL and WalkSAT algorithms